PHYSICAL REVIEW
LETTERS

VOLUME 4 APRIL 15, 1960 NUMBER 8

ATTEMPTS TO DETECT RESONANCE SCATTERING IN Zn^{67}; THE EFFECT OF ZERO-POINT VIBRATIONS*

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(Received March 23, 1960)

The 93-kev γ ray from the 9.4-μsec level of Zn^{67} would appear to offer an extreme example of the fractionally narrow γ rays in solids discovered by Mössbauer.[1] It has been suggested as an ideal medium for studying such small frequency shifts as would result from, for example, gravity.[2,3] We wish to describe some unsuccessful attempts to detect resonant absorption of this γ ray and to offer an explanation which has more general applicability and complicates the extension of the technique much beyond the narrowness presently achievable with Fe^{57}.[4-7]

The 93-kev γ ray is emitted as the final step in the decay of 78-hr Ga^{67} to stable Zn^{67}. Ga^{67} was prepared with good radiochemical purity by $(\alpha, 2n)$ reaction in Cu^{65} by about 10 microampere hours of bombardment by the MIT cyclotron.

Resonant absorption experiments were attempted for one combination of source and absorber at the temperature of liquid nitrogen and, for this and three other combinations, at the temperature of liquid helium. The experiments at helium temperature used a cryostat with a thin bottom to allow the 93-kev γ ray to be measured externally. In all experiments comparison was made of the γ-ray intensity transmitted through the absorber with the source stationary to that with it vibrating at an rms velocity of about 10^{-3} cm/sec, about one hundred times larger than the expected linewidth. The vibration was produced by a quartz electromechanical transducer which was also cooled. In all cases a null effect was found to the accuracy of the statistical uncertainty of $\pm 0.1\%$ of the counts in one channel. Correcting for background with the tenfold nonresonant absorption normally present in the absorbers leaves the result that no resonant absorption greater than 0.3% was present in any of the combinations. The combinations tried were:

1. Source Ga^{67} dissolved in 95% aluminum 5% by weight zinc alloy, and absorber 90% aluminum 10% zinc by weight, 4 cm thick. It was hoped that the aluminum would provide a cubic environment to avoid electric quadrupole splitting as well as a moderately high effective Debye temperature. Unfortunately the necessarily high zinc concentration probably destroys lattice symmetry seriously enough to broaden the lines through the inhomogeneous electric quadrupole interaction. The 11-kc/sec linewidth in the absorber resulting from the 9.4-μsec half-life is comparable to widths obtained in nuclear paramagnetic resonance and it is known that even 0.5% of zinc broadens the Al magnetic resonance line.[8] One would not expect the Zn to be less affected. Ignoring that broadening and supposing, optimistically, that the 375°K Debye temperature of the Al host applied to the recoil dynamics of the zinc, one would expect an effect of 0.4% at nitrogen temperature and 2.2% at helium temperature.

2. Source Ga^{67} dissolved in polycrystalline zinc and absorber 5-mm thickness of polycrystalline zinc. In this case one would expect a reduction of the cross section by a factor of 13/50 because of electric quadrupole splitting in the hexagonal close-packed lattice. The Debye tem-

perature of zinc is strongly temperature dependent. If one uses a value of 240°K, one expects only 0.08% effect, but if one uses the value of 305°K characteristic of the specific heat at 4°K or of the elastic constants, one expects 0.56%. Undoubtedly this estimate is overly optimistic because the low-temperature specific heat measurements and the elastic constants relate only to the low-frequency end of the spectrum of lattice waves whereas the high-frequency part dominates the recoil dynamics.

3. Source Ga^{67} dissolved into a single crystal of zinc with a 5-mm-thick single zinc crystal absorber. These crystals were oriented with c axes perpendicular to the direction of propagation of the γ rays observed and parallel to one another. It was hoped that the relative hardness of zinc in the direction perpendicular to the c axis would enhance the recoil-free component of the γ ray above that predicted from the Debye temperature.[9] Again, this is mainly connected with the low-frequency part of the phonon spectrum. In addition, the quadrupole splitting would reduce the cross section less with oriented single crystals than with polycrystalline samples.

4. Source Ga^{67} left in the cubic copper foil but annealed to relieve radiation damage, and about one-centimeter thickness of natural sphalerite, ZnS, from Franklin, New Jersey, reputedly of relatively high purity, used as absorber. In this case broadening and reduction of cross section by electric quadrupole coupling should have been avoided if the sphalerite was adequately pure. The moderately high Debye temperature of Cu should prove helpful. Using 315°K for the Debye temperature of copper and 270°K for ZnS leads to an expected effect of 1.15%.

The failure of the above attempts may involve several causes. The effects of the electron capture that converts Ga^{67} to Zn^{67} and of the γ rays leading to the 9.4-μsec state may not be healed adequately quickly and completely at helium temperatures to avoid broadening by electric quadrupole interactions. More fundamental, however, is an effect related to that involved in the predicted[10,11] and observed[10] variation with temperature of the frequency of the γ ray of Fe^{57}. The fractional width of the unbroadened line in Zn^{67} should be only 10^{-15}. It is now clear that the lattice energy shifts the frequency of the γ ray through the second order Doppler effect and that when different lattices are used for source and absorber the γ-ray and absorption line will be at different frequencies. Large shifts compared to 10^{-15} have been found between source and absorbers of Fe^{57} even when both are at a common temperature and ostensibly the same material.[12] Differences of Debye temperature were suggested as the cause. Differences of Debye temperature should result in such relative shifts even at the absolute zero of temperature owing to the difference in zero-point energies. Assuming a Debye spectrum for the lattice oscillators, the zero-point energy of $9k\theta_D/8$ per atom leads for Zn^{67} to a fractional decrease of frequency of 0.772×10^{-15} per degree increase in Debye temperature. If the zero-point vibration of the Zn^{67} in copper has an effective Debye temperature 45°K higher than that of the Zn^{67} in ZnS, then the absorption line in the ZnS would be fractionally 35 full linewidths higher in frequency.

Resonant absorption could be observed only by compensating that shift by a constant relative velocity. To find the line without prior knowledge of the displacement would require an extensive search and a good velocity spectrometer. Furthermore, inhomogeneity of the lattice could introduce important broadening and consequent weakening of the lines through small variations of the zero-point energies. Only in the experiments using zinc directly, of those reported above, would there appear to have been hope of finding absorption by the method used. For those cases the effect expected was probably too small to be observed.

Addition of the effect of the zero-point vibrations to the shifts induced thermally modifies the values of the differences in θ_D needed to explain the shifts at a common temperature between absorber and source reported previously for Fe^{57}.[12]

We wish to thank Mr. J. I. Musher for assistance with some of the experiments.

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*Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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SEARCH FOR THE ANISOTROPY OF INERTIA USING THE MÖSSBAUER EFFECT IN Fe^{57}†

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(Received March 18, 1960)

The width of the central resonance absorption line in the Mössbauer effect in Fe^{57} has been found by several observers to be considerably greater than the natural linewidth.[1-4] In all these cases the emitting and the absorbing nuclei were randomly oriented, and the possibility arises that this widening is due to an external perturbation which affects the nuclear levels.

An intriguing mechanism for line broadening has been pointed out by Cocconi and Salpeter.[5,6] The locally asymmetric distribution of matter in the universe, in accordance with Mach's principle, may cause a local anisotropy $\Delta M = \frac{2}{3}(M' - M'')$ in inertia. Here M' is the inertial mass for matter accelerated towards the center of the Galaxy and M'' the inertial mass for acceleration perpendicular to this direction. Such an anisotropy ΔM would give rise to a shift in atomic[5] and nuclear[6] energy levels. In the case of Fe^{57}, the shift would occur only for the excited state with spin 3/2. Cocconi and Salpeter compute the level shift ΔE to be $(\Delta M/M)\overline{T}\overline{P}_2$, where $\overline{T}$ is the average kinetic energy of the nucleon responsible for the transition, and $\overline{P}_2$ has the value 1/5 for the most favorable relative orientation between the nuclear angular momentum and the line to the galactic center. Assuming $\overline{T} = 10$ Mev and a line broadening of 10^{-8} ev, Cocconi and Salpeter conclude that $\Delta M/M$ has an upper limit of about 1 part in 10^{14}.

Since the excess broadening of the center resonance line is large, it is easy to determine whether it arises from effects due to the Galaxy. In our first test, we placed both the Fe^{57} source and the Fe absorber in parallel magnetic fields of about 1000 gauss. In this condition, the atomic magnetic fields at the emitting and absorbing nuclei are parallel and the postulated galactic shifts should be the same for each of the corresponding magnetic sublevels in source and absorber. Thus any line broadening due to anisotropic inertia should disappear. The experiment showed, however, that over a twenty-four hour period there was no significant change in the linewidth between the two conditions: (1) no magnetic field, random domain alignments, and (2) source and absorber in parallel magnetic fields. The accuracy of this test is limited by the fact that, although the domains are aligned, the local magnetic fields at the nuclei in the source and absorber are different, producing an additional broadening effect. The results showed clearly that the majority of the excess broadening was not due to the Galaxy.

To perform a more sensitive test for a possible galactic effect, a magnetic field of about 600 gauss at the moving horizontal absorber[3] was aligned in the north-south direction. The source was placed in a magnetic field of similar magnitude, but lying in a vertical, north-south plane, and oriented at +45° with respect to the horizontal (see Fig. 1). Each sidereal day, the galactic center rotates once around a line parallel to the earth's axis, changing its direction with respect to the magnetic fields of source and absorber. Cocconi and Salpeter assume that the level shift reaches an extreme value when the magnetic field H is perpendicular to the line toward the galactic center, and again when it is parallel to that direction. At an intermediate angle the level shift vanishes.

In the experimental arrangement, shown schematically in Fig. 1, there is an interval of about 10 hours, centered at 17:30 sidereal time, when the line toward the galactic center makes nearly equal angles with the two magnetic field direc-

tions. Under these conditions, no relative shift is expected in the upper state energy levels between the source and the absorber. In contrast, there is about a 3-hour interval, centered at 06:30 sidereal time, when the galactic line is nearly 90° with respect to the absorber, but nearly 45° with respect to the source. During this latter period, the source should be essentially unaffected by the Galaxy, but the energy levels of the upper state in the absorber should experience their maximum shift. With the Galaxy at the nadir, the shape of the center line should change, since it splits into two lines nearly symmetrically displaced in frequency, but whose intensities have a ratio of about 2:1.

We have made a number of runs through a complete sidereal cycle using the arrangement of Fig. 1 looking for periodic variation in (a) the amplitude of the center of the line, and (b) the amplitude of four Doppler-shifted points on the side of the line. We also examined the shape of the entire center line at various sidereal times to see if there were any obvious deviations from symmetry.

In addition, we made observations with the angle ϕ of Fig. 1 at -45°. For this arrangement, the line to the Galaxy makes equal angles with the two magnetic fields twice a day (at about 13:00 and 22:00, sidereal time), when no relative shift between the excited levels of the source and the absorber is expected, but a near-maximum relative shift is expected when the Galaxy is near the nadir (05:00) and also when it is due south (17:30).

The experiments were performed during the period March 2 through March 12, 1960, at lat. 40°N, long. 88°W.

Using the arrangement of Fig. 1 for $\phi = 45°$ and for $\phi = -45°$ the amplitude of the center point of the absorption line was constant as a function of sidereal time to within the expected statistical fluctuations of ±0.3% with the confidence level of 40% by the χ^2 test. Similarly, the amplitude at the half-intensity point was constant to within ±0.3% with a confidence level of 25%.

As an additional check, we looked for fluctuations in the amplitude of the center point of the line for the case where the magnetic fields were parallel, both horizontal and pointing north, $\phi = 0$. For these conditions no variation is expected and no significant time variation was found.

With the experimental linewidth of 1.2×10^{-8} ev, a splitting of the center line of magnitude

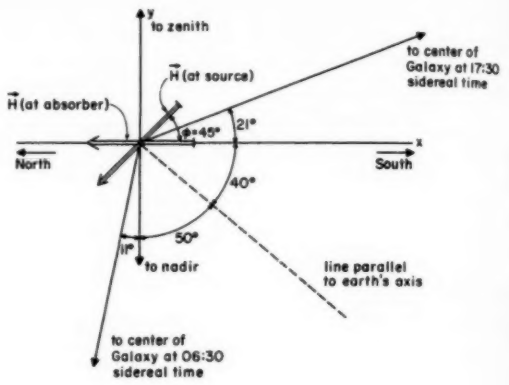

FIG. 1. The geometrical relationships between the magnetic fields on the source and the absorber with respect to the direction to the center of the Galaxy. Tests were also made with $\phi = -45°$.

$\Delta E = 1\times10^{-9}$ ev would have resulted in a clearly observable sidereal variation. Thus, since $E = 14.4$ kev, $\Delta E/E$ is less than 1×10^{-13}.

In order to calculate $\Delta M/M$ from $\Delta E/E$ certain assumptions must be made. Following Cocconi and Salpeter, and taking $\overline{T} = 10$ Mev and $\overline{P}_2 = 1/5$, we get the limit $\Delta M/M < 5\times10^{-16}$. Both assumptions are, however, open to debate, especially since the magnetic moment of the ground state[7] and of the excited state[8] of Fe^{57} are considerably smaller than one would expect from the shell model.

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†Supported in part by the joint program of the Office of Naval Research, the U. S. Atomic Energy Commission, and by the U. S. Air Force through the Air Force Office of Scientific Research and through the tri-Service contract with the Coordinated Sciences Laboratory administered by the U. S. Army Signal Corps.

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HIGH-FREQUENCY STUDIES ON SUPERCONDUCTING TIN*

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Microwave studies of normal metals both with and without a static magnetic field have provided information on the shape of the Fermi surface[1] and on the effective masses of the electrons.[2] In this note, the surface impedance measurements at 1 kMc/sec on superconducting tin[3] both with and without a magnetic field are analyzed to study changes in the band structure which accompany the transition from the normal to superconducting metal. It is found that for superconductors the zero-field experiments are relatively insensitive to the band structure as compared with experiments on the field variation of the surface impedance. Secondly, it is suggested that different carriers are important in the two problems, with the "heavy" electrons dominating the zero-field measurements, and the "light" electrons more important in the field-dependent studies.

The recent extension[4] of the Serber two-fluid model calculation of the superconducting surface impedance Z to include the term proportional to H_0^2 in $Z(H_0)$ allows an analysis to be made of the data for $H_0>0$, H_0 being the static magnetic field at the surface. The temperature dependence of the normal and superconducting electron concentrations in the superconducting state is taken to be that predicted by the Gorter-Casimir two-fluid model, i.e., $N_n=N_0t^4$ and $N_S=N_0(1-t^4)$, in which N_0, N_n, and N_S are the total, normal, and superconducting electron concentrations, respectively, and $t=T/T_c$ is the reduced temperature.

For both $H_0=0$ and $0<H_0<H_c$, H_c being the critical field at the temperature T, the Serber two-fluid treatment predicts the magnitude and temperature dependence of Z in terms of the parameters (m^*v_F) and λ_0, which characterize a given specimen. The quantities m^* and v_F are the effective mass and Fermi velocity of the normal electrons in the superconducting state and λ_0 is the penetration depth λ of the dc magnetic field evaluated at $T=0°K$. These quantities determine a dimensionless parameter α_0 given by

$$\alpha_0=\frac{\lambda_0}{\delta_n}=\lambda_0\left(\frac{3\pi^2\omega N_0e^2}{m^*v_Fc^2}\right)^{1/3}, \tag{1}$$

in which δ_n is the superconducting skin depth for the rf fields in the extreme anomalous limit appropriate to the total electron concentration N_0. The quantities N_0 and λ_0 are known from other measurements. The temperature dependence of α_0 is attributed to the variation of (m^*v_F) with T. The value of $\alpha_0=0.16$ is appropriate for Sn, if $m^*=m_0$ (free electron mass), $v_F=6.6\times10^7$ cm/sec (normal state value)[5] and $\lambda_0=5.0\times10^{-5}$ cm.[6]

The determination of α_0 for a single-crystal tin sample[3] was made for both $H_0=0$ and $H_0>0$. For comparison, the field variation of Z was also measured and analyzed for the same sample in the normal state.[7] Figure 1 shows a reduced temperature plot of the zero-field surface re-

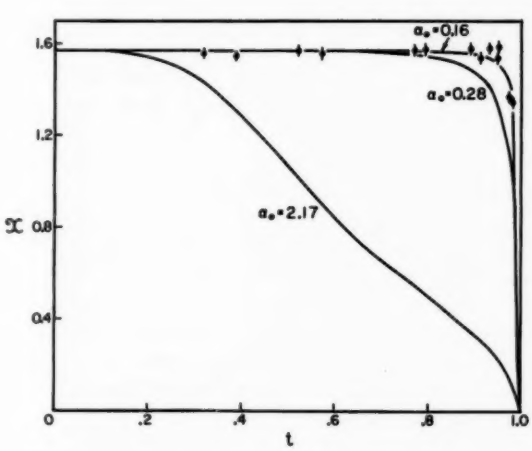

FIG. 1. Plot showing zero-field surface reactance vs reduced temperature. The solid curves give the dimensionless quantity $\mathfrak{X}=[c^2\mathfrak{X}(0)/16\pi\nu\lambda]$ as calculated from the two-fluid model for three values of α_0 (see text). The experimental points are for a single-crystal tin sample at 1 kMc/sec.

actance $X(0)$ in dimensionless units, $\mathfrak{X} = [c^2 X(0)/16\pi\nu\lambda]$, as calculated from Serber's model for $\alpha_0 = 0.16$, 0.28, and 2.17. The quantities ν and c are the frequency and velocity of light, respectively. It is seen that the experimental points best fit $\alpha_0 = 0.16$. For comparison, surface impedance measurements on this sample in the normal state yield an effective mass $(m^*/m_0) = 0.6$ and a relaxation time $\tau = 9\times10^{-11}$ sec. The curve $\alpha_0 = 0.28$ in Fig. 1 was determined from the measurements of the frequency dependence of the surface resistance of Sn by Kaplan, Nethercot, and Boorse[8] in the frequency range between 17 and 77 kMc/sec. The disagreement between the curve for $\alpha_0 = 0.28$ and the experimental points indicates that the two-fluid model analysis used here is not applicable to the mm frequency range. However, at frequencies $\nu < 1.5$ kMc/sec a two-fluid model is expected to be adequate for Sn.[9]

Figure 2 shows a theoretical plot as a function of reduced temperature of the longitudinal field variation of the surface reactance $\Delta X_L(H_0) = [X_L(H_0) - X(0)]$ for $\alpha_0 = 2.17$, which fits the experimental points in the region $t < 0.7$. At higher temperatures, $t > 0.7$, the data are fitted by $\alpha_0 = 5$. In terms of the parameter $\gamma = (m^* v_F/m_0 v_F')$, v_F' being the Fermi velocity in the normal state, $\alpha_0 = 2.17$ and $\alpha_0 = 5$ correspond to $\gamma = 3.8\times10^{-4}$ and $\gamma = 3\times10^{-5}$, respectively.

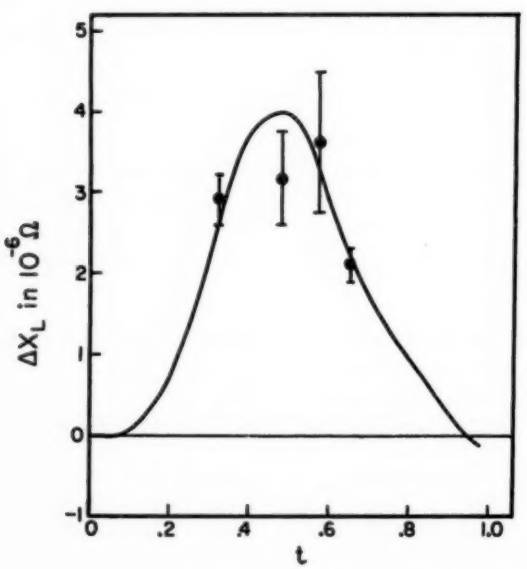

FIG. 2. Plot of field variation of surface reactance vs reduced temperature. The solid curve is calculated for $\alpha_0 = 2.17$. The experimental points give the term in H_0^2 of $\Delta X_L(H_0)$ for a tin sample at 1 kMc/sec, $t < 0.7$. ΔX_L corresponding to $\alpha_0 = 0.16$ is too small by a factor 10^{-7}. The value of H_0 used to evaluate $\Delta X_L(H_0)$ is the critical field at the temperature t.

In the presence of H_0, both the magnitude and the temperature dependence of $\Delta X_L(H_0)$ are determined by $(m^* v_F)$. The magnitude of $\Delta X_L(H_0)$ is particularly sensitive to the choice of α_0, with $\Delta X_L(H_0) \propto \alpha_0$.[6] On the other hand, the zero-field experiments are relatively insensitive, α_0 being determined by the temperature at which $\mathfrak{X}$ departs from the constant value of $\pi/2$. To emphasize the fact that different values of α_0 are needed to fit the data for $H_0 = 0$ as opposed to $H_0 > 0$, a curve of $\mathfrak{X}$ is plotted in Fig. 1 for $\alpha_0 = 2.17$. Furthermore, the parameter $\alpha_0 = 0.16$ which fits the zero-field data yields a value of ΔX_L too small by a factor of 10^7.

Anisotropy in the band structure is indicated by a comparison between the longitudinal and transverse field experiments. The transverse field data for X are fitted by $\alpha_0 = 2.29$ in the low-temperature range and $\alpha_0 \cong 6$ in the temperature range near T_c.

A comparison between theory and experiment for the longitudinal field variation of the surface resistance $\Delta R_L(H_0)$ gives the correct amplitude of the effect with the same choice $\alpha_0 = 2.17$, but not the correct shape. However, no other value of α_0 (or γ) would give a better fit for the data; if α_0 is chosen to fit the shape better, the amplitude of $\Delta R_L(H_0)$ is too small by several orders of magnitude. Thus, a real defect in our present two-fluid model seems to be indicated. Miller has shown that nonlocal corrections to the London equations are important for R, but less important for X.[10] It thus seems likely that a nonlocal theory for the superconducting electrons is indicated, at least for $\Delta R_L(H_0)$.

The above results seem to indicate that the excitations above the superconducting state are of several sorts. There are "heavy" electrons and some "lighter" electrons. Nearly all of the excited electrons are "heavy" electrons. The "heavy" electrons make the major contribution to $R(0)$, $X(0)$, specific heat, thermal conductivity, and ultrasonic attenuation experiments. The "lighter" electrons, being far less numerous, have only a small effect on these experiments. However, the magnetic field experiments emphasize the contributions of the "light" electrons by the weighting factor α_0.[6] Evidence that these "light" electrons are connected with the super-

conducting state comes from the absence of "light" electrons in the de Haas–van Alphen[11] and cyclotron resonance[12] measurements on normal tin. At low temperatures (e.g., $t<0.7$) one band of "light" electrons is excited ($\alpha_0=2.17$), while near T_c a second band appears ($\alpha_0=5$). As soon as the second band of "lighter" electrons is excited, their effect completely dominates $\Delta X_L(H_0)$. The apparent temperature independence of α_0 is compatible with an exponential E vs k for the "light" bands of normal electrons.

It is hoped that a more detailed experimental study will be able to decide upon the electronic band structure of the superconducting state.

The authors wish to acknowledge the hospitality of the Institute for the Study of Metals, University of Chicago, where the measurements were made.

*Supported in part by the Office of Naval Research.
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MEASUREMENT OF LOCAL FIELDS AT IMPURITY Fe^{57} ATOMS USING THE MÖSSBAUER EFFECT

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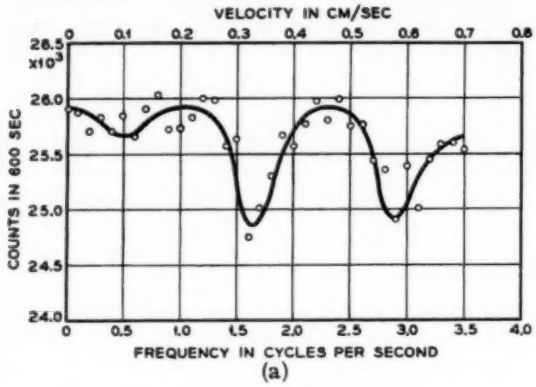
(Received March 21, 1960; revised manuscript received March 25, 1960)

The Mössbauer effect[1] of Fe^{57}[2] has proved to be a convenient tool for the investigation of local magnetic fields in the transition metals, where other methods have not been entirely successful. We have investigated the local fields in Fe, Co, and Ni, as well as in a number of other substances.

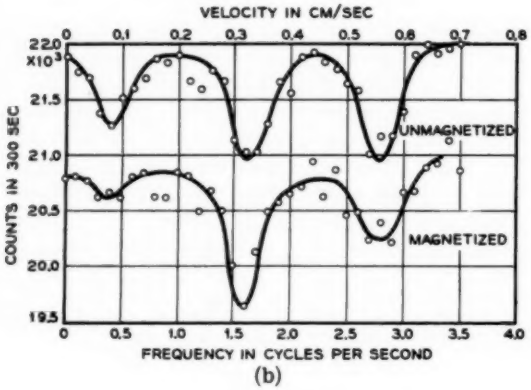
The sources we used were prepared in a number of different ways. A source of Co^{57} in metallic iron was prepared by electroplating from a carrier-free $CoCl_2$ solution onto a high-purity iron foil 0.0025 cm in thickness, and then diffusing at 950°C in a hydrogen atmosphere. Sources in metallic cobalt and nickel were prepared by evaporating some of the $CoCl_2$ solution to dryness on a foil of the metal and diffusing as before. Each source was then mounted on a loudspeaker voice coil driven at constant velocity with a symmetrical sawtooth wave of such magnitude as to produce a 1-mm excursion of the coil. No distinction was made between positive and negative velocity, and counting was continued through turn-around.

As an absorber we have most commonly used a 0.0025-cm foil of Type 310 stainless steel (25% chromium, 20% nickel). This material, although paramagnetic, exhibits an unsplit absorption line, indicating that the spin correlation time τ is sufficiently short to satisfy the relationship $\tau A/\hbar \ll 1$, where A is the hyperfine coupling of the Fe^{57} nucleus. We have also used diamagnetic potassium ferrocyanide which, as expected, also exhibits an unsplit absorption line. The salt $K_4Fe(CN)_6\cdot 3H_2O$ was crushed, mixed with a small amount of binder, and formed into a uniform sheet 0.08 cm thick. This thickness was chosen to make the sheet contain the same amount of Fe^{57} per unit area as a metallic iron foil 0.0025 cm thick. However, the high density of foreign matter makes this absorber less attractive than the stainless steel.

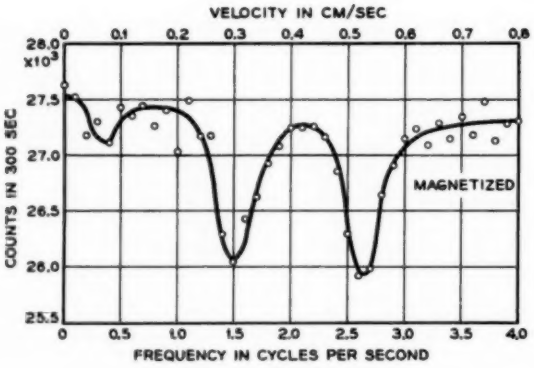
Results obtained with the iron source at room temperature and the potassium ferrocyanide absorber at 78°K, Fig. 1(a), directly exhibit the hyperfine structure of Fe^{57}. The three absorption lines yield values for the nuclear g factor for the



(a)



(b)



(c)

FIG. 1. (a) The hyperfine structure of Fe^{57} in metallic iron obtained with a potassium ferrocyanide absorber. (b) The hyperfine structure of Fe^{57} in metallic iron obtained with a stainless steel absorber. (c) The hyperfine structure of Fe^{57} in metallic cobalt obtained with a stainless steel absorber.

first excited state and for the magnetic field at the nucleus which are in good agreement with the earlier determination by the Argonne group.[3] The observed effect is reduced by 35% if the absorber is at 300°K. Similar results obtained with the stainless steel absorber, for the cases of magnetized and unmagnetized iron foil source, are shown in Fig. 1(b). The relative intensity of the three lines indicates that there is some remanent magnetization in the source which strengthens the middle hyperfine line even in the absence of an external magnetic field. Figure 1(c) shows the results obtained when the iron foil source is replaced by a cobalt metal foil. The small shift in the location of the hyperfine lines indicates that the magnetic field at the Fe^{57} nucleus in cobalt metal is 3.1×10^5 oe, which may be compared to the value of 3.3×10^5 oe in iron metal. In nickel, at room temperature where the magnetization is 0.95 of the saturation value, the corresponding field is 2.6×10^5 oe. A more detailed investigation of this question is in progress.

The linewidths in the stainless steel experiment are significantly larger than those obtained in the original Fe^{57} experiment,[2] in which both source and absorber were iron metal. The earlier experiment has been duplicated with our equipment, yielding results in agreement with those in the literature. Line broadening could arise in the stainless steel experiments from a spin correlation time which is insufficiently short. If this were the dominant effect it should be possible to obtain a measure of the spin correlation time in the stainless steel through the relation $\Delta\nu > 2\tau I^2 A^2/\hbar^2$. The observed line width, $\Delta\nu$, corresponds to a correlation time $\tau \leq 10^{-10}$ sec. On the other hand, the line broadening might also arise from the combining of positive and negative velocities in the experimental setup, provided there is a shift in the energy of the emitted gamma ray relative to resonant energy of the absorber. Such shifts may be due to the second order Doppler effects[4] or due to differences in the environment of the iron nuclei in the source and in the absorber. We have observed a shift of this nature for Fe^{57} in Ge.

In addition to our experiments with transition metals, we have also observed the Mössbauer effect of Fe^{57} in n-type silicon. The source was prepared in a manner similar to that described for the cobalt and nickel foils above; as a final step, it was etched in hydrofluoric acid to remove any oxide layer which might contain Co^{57} in a different environment. Consistent results were obtained with both the stainless steel and the potassium ferrocyanide absorbers. The data in Fig. 2 show that the hyperfine field at an iron nucleus occupying a site characteristic of a cobalt atom in silicon is approximately 3×10^4 oe. This value is very much smaller than that found

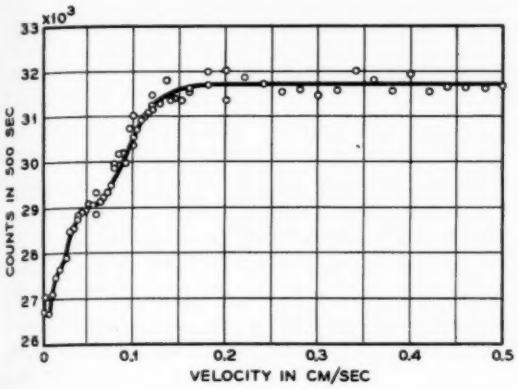


FIG. 2. The hyperfine structure of Fe^{57} in n-type silicon obtained with a stainless steel absorber.

for iron in a normal iron site.[5] The silicon source exhibits sufficiently weak hyperfine coupling so that samples of this type may be useful as unsplit sources in the investigation of other iron-bearing materials. They have the advantage over sources made by incorporating Co^{57} into stainless steel or potassium ferrocyanide that self-absorption in the source is negligible due to the absence of stable Fe^{57}.

The author is indebted to W. L. Brown, V. Jaccarino, and W. E. Blumberg for discussions and suggestions during the course of this work; to K. M. Olsen for the high-purity iron, nickel, and stainless steel foils used in these investigations; and to H. C. Montgomery, G. Feher, and J. Eisinger for the loan of numerous pieces of equipment.

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RECOILLESS RAYLEIGH SCATTERING IN SOLIDS

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(Received March 21, 1960)

Using the Mössbauer effect, photon sources and analyzers extremely selective in energy are now available. We study here with such an analyzer the recoilless Rayleigh scattering by atoms in solids.

This effect is related to the x-ray diffraction by crystals as follows. The interference at the exact Bragg angles occurs when the scattering is elastic with respect to the lattice as a whole, that is, without any phonon exchange. Debye and Waller have calculated the reduction in intensity of x rays scattered at the Bragg angles in a solid at temperature T,[1]

$$\varphi_T = \exp\left\{-\frac{3}{2}\frac{E_R}{k\theta}\left[\frac{1}{4}+\frac{1}{x}\int_0^x \frac{u\,du}{e^u-1}\right]\right\}, \qquad (1)$$

where $x = T/\theta$, θ is the Debye temperature, and $E_R = (E^2/Mc^2)(1-\cos\theta)$ is the recoil energy given to the free atom by a photon of energy E scattered at the angle θ.

In the present work, where we detect the elastic scattering directly by an energy selection instead of analyzing a diffraction pattern, the factor φ_T is the relative number of photons scattered without energy change. It is clearly the same factor which gives the proportion of recoilless γ rays in the Mössbauer effect[2]; in that case $E_R = E^2/2Mc^2$ in Eq. (1).

In order to measure the factor φ_T, we have studied the Rayleigh scattering for several materials: Pt, Al, graphite, and paraffin. The 23.8-kev photons emitted by Sn^{119*} were scattered at $50° \pm 5°$ and absorbed by a Sn^{119} foil 40 mg cm^{-2} thick (almost completely black for the recoilless photons[3]) (Fig. 1). The scatterers' thicknesses were such that the transmission of the γ rays was of the order of 10%.

The Rayleigh-scattered photons are accompanied by inelastically scattered photons (Raman, Compton), considerably shifted in energy, so that the selective absorption in Sn^{119} occurs only

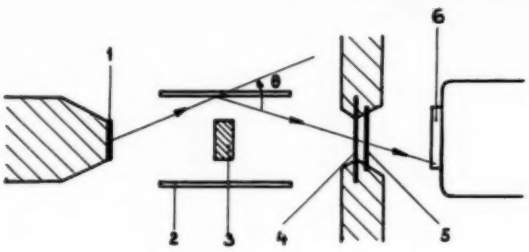

FIG. 1. (1) Sn^{119*} source; (2) scatterer; (3) bismuth stopper; (4) 40 mg cm^{-2} Sn^{119} foil (71.5% Sn^{119}); (5) 62 mg cm^{-2} Pd foil absorbing Sn x rays; (6) 1.5 mm NaI(Tl) scintillator and photomultiplier.

for a fraction α of all the scattering processes; α is extracted from the form factors given by Compton and Allison.[4]

We have measured the relative decrease Δ of counting rate between room temperature $T_1 = 300°K$ and $T_2 = 80°K$. The recoilless scattering proportion at T_2 is approximately

$$\varphi_{T_2} = \frac{1}{\alpha}\,\frac{\Delta - \epsilon}{f_2 - \epsilon},$$

where f_2 is the ratio of recoilless emission of the Sn^{119*} source and ϵ is its self-absorption at T_2. Here $f_2 = 0.32 \pm 0.015$ and $\epsilon = 0.05 \pm 0.01$. We neglect the small recoilless emission at 300°K which introduces a negligible correction for φ_{T_2}.

The results are given in Table I. The agreement between the calculated and experimental values of φ is reasonably good, especially when

Table I. Experimental and calculated values of φ_{T_2}.

	$1/\alpha$	Δ	φ_{T_2}	φ_{calc}
Pt	1.05	0.27 ± 0.03	0.72 ± 0.09	0.80
Al	2.15	0.19 ± 0.016	0.92 ± 0.09	0.62
C	4.13	0.10 ± 0.01	0.79 ± 0.09	0.68
CH_2	5.30	0.020 ± 0.01	0	

we notice that the Debye temperatures are deduced from specific heat measurements rather than from x-ray diffraction.

We have also, using a thin Sn^{119} foil as a scatterer, observed at low temperature the resonant Mössbauer scattering.[5]

This method extends the range of solids which can be studied by means of the Mössbauer effect or by x-ray diffraction.

It is a pleasure to acknowledge interesting discussions with Dr. Abragam, Dr. Cotton, and Dr. Jacrot.

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STUDY OF THE INTERMEDIATE STATE IN SUPERCONDUCTORS USING CERIUM PHOSPHATE GLASS

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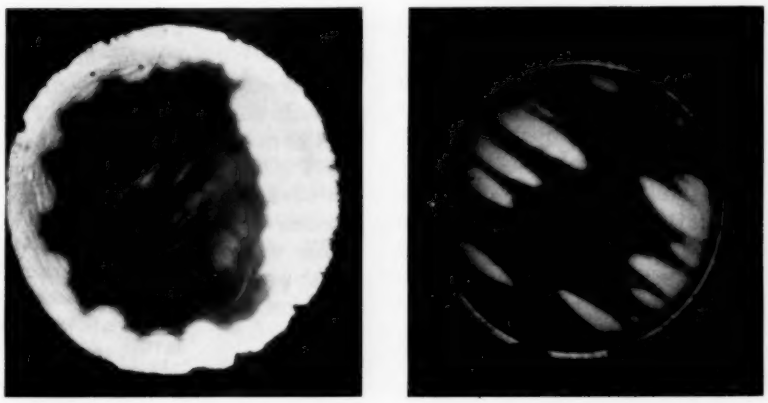
Alers[1] has recently used the magneto-optic Faraday rotation in some cerous nitrate-glycerol solutions to observe the intermediate state in a superconducting lead alloy. This technique, like the bismuth probe[2] and the superconducting[3,4] or ferromagnetic[5] powder methods, makes use of the diamagnetism of the superconducting state to investigate the topography of the superconducting and normal domains at the surface of a specimen. In all these methods, it is assumed that the local magnetic field distribution and variations at the surface reflect the amount of normal and superconducting material in the specimen itself. More recently, Alers[6] has reported large Faraday rotations in some paramagnetic glasses containing cerium meta-phosphate suggesting their use in studying the structure of the intermediate state in superconductors.

Utilizing this Faraday effect in similar glass prepared by A. Pincus and R. H. Pry of this laboratory, we have obtained some preliminary results on resolving the intermediate state in various superconducting metals. Included in these early studies was the effect of different

conditions of metallurgy, strain, purity, crystallographic orientation, grain size, etc. The purpose of this note is to point out that new information on the dynamics and time-dependent phenomena of the intermediate state not previously seen by other techniques may now be observed. Because still photographs do not demonstrate the full power of this method, motion pictures are more useful in recording the kinetic details; e.g., wall motions, annealing behavior of frozen-in flux, etc. The technique also facilitates studies involving both the magnitude and the direction of frozen-in moments (not feasible by powder techniques), supercooling phenomena and critical field determinations.

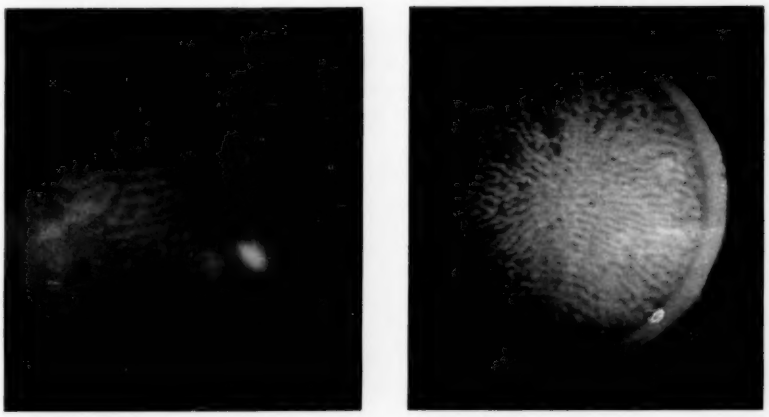
The success in the resolution (better than 0.2 mm) is due in part to the use of very thin glass (~0.25 mm thick). Photographs illustrating this are presented in Figs. 1 and 2. Light areas are normal and the dark regions are superconducting. The monochromatic light beam used (5451 A) was obtained from a Hg-arc source. The transverse magnetic field was provided by a superconducting niobium wire-wound solenoid.

Figure 1(a) shows the intermediate-state structure of a polycrystalline tantalum disk (7/8-inch diameter $\times 1/16$ inch thick), cold-worked by compression after machining from a cast ingot. Figure 1(b) presents the intermediate-state pattern of a cold-rolled 7/8-inch diameter disk; the disk was cut from a polycrystalline tantalum sheet 0.009 inch thick produced by cold-rolling of an "as cast" ingot of vacuum arc-melted tantalum. The residual resistance ratios of each of these two materials were comparable (i.e., $R_{273°K}/R_{4.2°K} = 12$). These patterns could be



(a) (b)

FIG. 1. (a) Intermediate-state pattern of a tantalum disk (cold-worked) at 1.46°K. H_{applied} =460 oersteds. (H_C = 1060 oersteds.) (b) Intermediate-state pattern of a thin tantalum disk (cold-rolled) at 1.52°K. H_{applied} =164 oersteds. (H_C = 1030 oersteds.)



(a) (b)

FIG. 2. (a) Nonequilibrium pattern obtained on a rectangular plate of single-crystal zone-refined tin 10 minutes after transverse field had been slowly reduced from 222 oersteds to 32 oersteds at T = 1.46°K. Pattern changes with time. (H_C = 215 oersteds.) (b) Nonequilibrium pattern obtained on a polycrystalline tin disk 21 minutes after transverse field had been reduced from 325 oersteds to 58 oersteds at T = 1.40°K. Pattern changes with time. (H_C = 264 oersteds.)

altered by subsequent heat treatments which were also accompanied by decreases in critical field. Irregular patterns similar to Fig. 1(b) have also been observed for a cold-rolled sheet of niobium. In each of these observations, as the field is raised slowly and uniformly from zero to its critical value, large and irregular-shaped normal domains appear very suddenly [see Fig. 1(a) and 1(b)]. These abrupt jumps of domain walls are similar to those observed by Schawlow[3] on a polycrystalline sample of vanadium and on a cold-worked sample of rhenium recently reported by Schawlow et. al.[7] with superconducting powder techniques. Experiments to elucidate the nature of these boundary motions are now in progress.

Figure 2(a) shows a domain pattern (nonequilibrium) evident in a zone-refined unstrained single crystal of tin (rectangular flat plate 1/2 inch × 1/2 inch × 7/8 inch) about 10 minutes after the applied field had been reduced from slightly above the critical field ($H_C = 215$ oersteds) down to about 32 oersteds at 1.46°K. To reduce the field abruptly to zero would result in similar patterns, appearing in a shorter time and annealing more rapidly; they, therefore, do not photograph as well. The corrugated pattern in Fig. 2(a) is similar, though not identical, to those observed by Balashova and Sharvin[5] on less pure monocrystalline tin spheres. Their observations were made at somewhat higher temperatures but were reported to have been made under equilibrium conditions. Equilibrium patterns of the intermediate state, in our case, reveal, for applied fields greater than about 150 oersteds, a single normal domain surrounded by a border area that is superconducting. The inner boundary of this area seems to move out uniformly with increasing applied field. The magnetic lines, presumably, pass through the superconducting area in small, unobservable channels. The details of these equilibrium patterns are not similar to Schawlow and Devlin's[8] observations on pure tin crystals, although the macroscopic anisotropic kinetic effects with increase in field observed by us are similar to their results.

Figure 2(b) shows another nonequilibrium pattern for a polycrystalline (1 mm average grain size) disk of tin (7/8-inch diameter × 0.270 inch). The sample was made from an ingot supplied by the Vulcan Detinning Company and listed as "extra pure fine grade" containing 0.0003% lead and 0.0001% iron. These patterns are similar to those reported by Schawlow[3]; however, details on their growth and annealing behavior have not been observed before.

Frozen-in flux has also been resolved in zone-refined single crystals of lead both in a strained and unstrained condition. The patterns are different in each case.

Details of the experimental technique and of the various results will be presented elsewhere.

The author has benefitted from discussions with many associates, in particular, with R. H. Pry for his suggestions and encouragement in the use of the glass and with C. P. Bean. He is indebted to D. T. F. Marple and S. Roberts for suggestions on the optical system. K. T. Aust and J. W. Rutter were kind enough to supply him with the various single crystals. W. A. Healy capably assisted in the experimental work.

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SPUTTERING THRESHOLDS AND DISPLACEMENT ENERGIES*

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(Received March 28, 1960)

Sputtering at very low ion energies (below 50 ev) is of interest in several areas, particularly with reference to the question of whether sputtering threshold energies might be linked to displacement energies in metals. Other reasons for interest are connected with radiation damage problems and the sputtering of satellites. The major obstacle to work at very low ion energies has been the lack of a sensitive and generally applicable method of measuring the small number of sputtered atoms.[1] Sensitive electrical detectors cannot be employed because sputtered atoms are neutral. Surface ionization[2] and radioactive tracer methods[3] have been used as sensitive indicators, but these methods are applicable only to a limited number of target materials. At very low sputtering yields another difficulty encountered is contamination of the target surface from background impurities in the tube.

A newly developed spectroscopic method eliminates most difficulties and permits sensitive relative yield measurements in the low ion energy region for any target material. Atoms are sputtered from a target in a magnetically concentrated low-pressure mercury or rare gas discharge. In mercury the discharge data are: gas pressure ~2 microns, discharge current between pool-type cathode[4] and anode ~4 amperes, and discharge voltage ~35 volts. The mercury pool-type cathode which is very convenient in a demountable tube can be retained for noble gas studies by freezing the mercury out of the main discharge section and supplying a noble gas instead. The discharge data in argon are: gas pressure ~7 microns, discharge current ~4 amperes, and discharge voltage ~50 volts. The sputtered atoms find very favorable excitation conditions (spiraling electrons) in the plasma and one observes the emission spectrum of the target atoms superimposed on the emission spectrum of the discharge gas. A strong emission line of the target material is monitored by use of a monochromator employing a photomultiplier detector for intensity measurement. The curve of the spectral line intensity plotted as a function of bombarding ion energy in most cases closely matches the yield curve obtained by absolute methods. The sensitivity of this method is such that one can readily measure yields done to 10^{-4} atom per ion for many metals. Difficulties with surface contamination of the target are overcome by incorporating in the tube a magnetically operated shutter to protect the observation window from sputtered deposits and by cleaning the target with high ion energy bombardment immediately before each measurement. In practice, the shutter is left closed and the target held at a high voltage continuously except for the few seconds needed for taking data.

Yield curves for sputtering by Hg^+ ions and A^+ ions have been obtained with this method for about twenty different metals. Typical curves with yields on a log scale are shown in Fig. 1. The following features are of interest in these results:

1. Yields at very low ion energy become sensitive to the anode voltage above a certain critical value. This is supposedly caused by the formation of doubly charged ions which arrive at the target with twice as much kinetic energy as singly charged ions.
2. With the contribution due to doubly charged

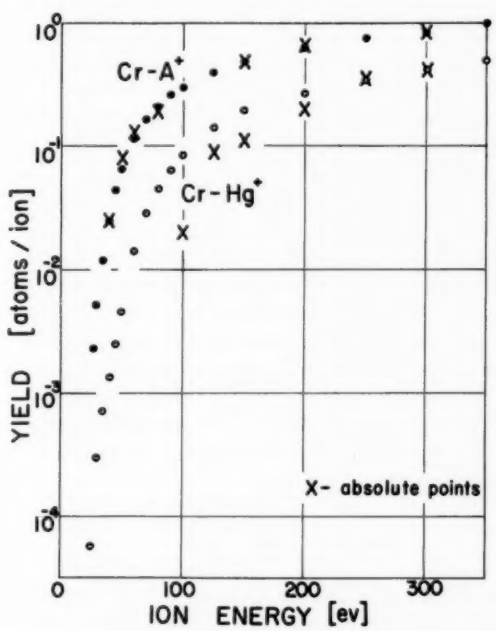

FIG. 1. Yield curves for chromium in argon and in mercury.

ions minimized by operating the tube with a low anode voltage, one finds that the yields decrease with decreasing ion energy more rapidly than previously reported. Thus, at least for normal ion incidence, sputtering effects at low ion energy seem to have been overestimated in the past.

3. By extrapolating the lower part of the yield curves, one finds that approximate threshold energies for sputtering are in the range 15 to 45 ev. For argon the product of these thresholds, and the energy transfer factor $4m_1m_2/(m_1+m_2)^2$, is with few exceptions (Pd, U, and Th ~ 15 ev) close to 25 ev which is the energy generally associated with the displacement threshold for radiation damage.[5] The measurements are presently being continued and extended to helium and other rare gases.

*Work partly supported by the U. S. Air Force Cambridge Research Center and the Office of Naval Research.

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PROPOSAL FOR AN ELECTRON SPIN RESONANCE EXPERIMENT OF S-STATE IONS UNDER HIGH HYDROSTATIC PRESSURE

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(Received March 28, 1960)

The author[1] has shown that a crystal field model gives the cubic field splitting parameter a for Mn^{2+} proportional, to a first approximation, to the square of the cubic crystal field. This reasoning may be extended to ions having the half-closed configuration l^{2l+1} and is explained essentially as follows. Two ions are said to be complementary when the patterns of their linear crystalline Stark splittings are inverted. This is the case for l^x and $l^{2(2l+1)-x}$. Now l^{2l+1} is complementary to itself. If its multiplet were split, it would show two inverted patterns. But since a given multiplet shows only one pattern, any multiplet from l^{2l+1} cannot be split by a linear action of the crystal field. It can be split to the next approximation by the higher order perturbation involving squared off-diagonal elements of the cubic potential. Thus the result may be extended to the l^{2l+1} configuration. The cubic field splitting parameter a would be of the form

$$a=\alpha(D_q)^2-\beta(D_q)^4, \qquad (1)$$

where α and β are constants, and D_q is a measure of the cubic field. In a first approximation only the nearest neighbor ions around the S-state ion considered may be assumed to contribute to D_q. Thus one obtains

$$D_q=\langle r^4\rangle(e_1e_2/R^5)(1/336)\times\sum_j(35\cos^4\theta_j-30\cos^2\theta_j+3), \qquad (2)$$

where $\langle r^4\rangle$ is the average of r^4 with respect to the radial part of the l orbital; R is the interionic distance between the S-state ion and neighbor ions possessing charges e_1 and e_2, respectively; θ_j is the azimuth angle of the jth ion with respect to the [001] direction, which is chosen as the z axis of polar coordinates; and j is summed over the nearest ions. It is concluded that the parameter a should be proportional to the inverse tenth power of R and should be positive regardless of the sign of D_q.

Several electron spin resonance (ESR) experiments support this result. Baker, Bleaney, and Hayes[2] reported that ESR experiments on Mn^{2+} in calcium fluoride yielded

$$a=(+0.6\pm0.4)\times10^{-4}\ \mathrm{cm}^{-1}.$$

On the other hand, using $D_q(\mathrm{MgO})=4.0\times D_q(\mathrm{CaF_2})$ and a for MgO ($a=+18.6\times10^{-4}\ \mathrm{cm}^{-1}$[3]), they found that from the author's graph[1] of a against D_q that a for CaF_2 should be $\sim+1\times10^{-4}\ \mathrm{cm}^{-1}$ which is in remarkably good agreement with their value.

Matarresse and Kikuchi[4] did an ESR experiment on Mn^{2+} in ZnS in which Mn^{2+} is presumed to be surrounded by a tetrahedron of four negative sulfur ions and the sign of D_q would be, therefore, negative. As a result, this system provides an important test for the author's theory. They reported a negative sign for a, which seemed to be inconclusive since this negative sign was determined without measurements of the relative intensities of the transitions at low temperature. Watkins[5] made such a measurement at 4°K and found that the sign reported was in error. He reported that the sign for a should be positive.

Geschwind[6] reported that the parameter a for Fe^{3+} is positive in both octahedral and tetrahedral O^{2-} coordination in single crystal of yttrium gallium garnet. Moreover, he reexamined the spectrum of Fe^{3+} in rubidium aluminum sulfate where a negative sign for a was reported earlier,[7] and found that a is positive in this case also. These results appear to confirm the author's theory.

Recently Low and Rosenburger[8] have observed the paramagnetic resonance spectra of Gd^{3+} in single crystals of strontium chloride and obtained

$$a\ (c \text{ in their notation}) = (\pm 39.6 \pm 0.1) \times 10^{-4}\ \text{cm}^{-1}.$$

On the other hand, Low[9] reported earlier that a for Gd^{3+} in calcium fluoride is $(+185 \pm 5) \times 10^{-4}$ cm^{-1}. The ratio of the values of a measured in CaF_2 and $SrCl_2$ is about 4.7. The ratio of the Sr-Cl and Ca-F distances is about 1.28-1.29. Assuming that the trivalent gadolinium ion does not change this ratio appreciably, they calculated the ratio of the values of a in CaF_2 and $SrCl_2$, obtaining about 11.8. This calculated ratio is larger than the experimental one. Thus they suspect that there may be a mechanism responsible for the splitting being linear in the potential. They also point out that another possibility is to assume that the Gd-Cl or Gd-F distances differ considerably from the Sr-Cl or Ca-F distances.

We wish to call attention to the value of making an ESR experiment on S-state ions under high hydrostatic pressure which reduces the interionic distance R and induces an increase of the parameter a. Such an experiment may provide an important test for the author's theory which predicts an inverse tenth power dependence of a on R. This dependence may be approximately replaced by an inverse ten-thirds dependence on the volume of a single crystal. If this dependence were confirmed, one may calculate the ratio of interionic distances between S-state ions, contained in the host crystal as impurity, and the surrounding ions in various crystals. This ratio may give important information about the host lattice deformation due to impurity ions. On the other hand, if an inverse fifth power dependence were found it would provide an important clue in the investigation of the mechanisms responsible for the splitting parameter a.

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EVIDENCE FOR QUADRUPOLE INTERACTION OF Fe^{57m}, AND INFLUENCE OF CHEMICAL BINDING ON NUCLEAR GAMMA-RAY ENERGY*

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(Received March 30, 1960)

The recoil-free emission and resonant absorption[1] of the 14.4-kev nuclear gamma ray of Fe^{57}, has been used to determine the quadrupole coupling for the 3/2- excited state of Fe^{57} bound in Fe_2O_3, and to measure an energy shift of this nuclear gamma ray which is attributed to effects of chemical binding. This effect is corollary to the effects of chemical environment on internal conversion coefficients[2] and on electron capture disintegration rates.[3] These measurements also yield the value of the internal magnetic field at the position of the Fe^{57} nucleus when it is bound in antiferromagnetic Fe_2O_3.

The $M1$ emission line of Fe^{57} bound in ordinary metallic iron is split into six components by the magnetic hyperfine interaction.[4] The resonant absorption of this emission spectrum by Fe^{57} bound in Fe_2O_3 has been examined, as well as the much simpler absorption pattern which results when the "unsplit" emission line from Fe^{57} bound in a stainless steel lattice[5] is used. Because the interpretation of the two sets of measurements agree, only the latter measurements are presented in this Letter. The former measurements, in which a Co^{57} source co-plated with iron onto 1-mil copper was used, will only briefly be remarked upon.

The ~2-mg/cm^2 Fe_2O_3 absorber used in these measurements contained Fe^{57} enriched to ~30%. The source consisted of Co^{57} plated onto 0.001-inch stainless steel (25% Cr, 20% Ni). After plating, this source was annealed for one hour at 900°C in a hydrogen atmosphere. All measurements were made with source and absorber at a temperature of 25°C.

In order to obtain a Doppler shift of the emission line, a uniform motion was obtained by coupling a pneumatically driven cylinder to another cylinder filled with oil, the ports of which were connected via a needle valve. A wide range of uniform velocities could be selected by adjusting this valve. The direction of source travel was reversed automatically by means of microswitches. Additional microswitches, set to exclude the region of nonuniform motion near the travel limits, were used to provide gate signals for the counters. The distance of travel between the limits of the counting gates was 0.973 cm. Source velocity was determined by counting the cycles from a 1000 cps tuning fork oscillator during the time between the gate limits.

The 14.4-kev gamma ray was detected with a NaI(Tl) scintillation counter. The phosphor was 2 mm thick and 1.5 inches in diameter. The counter face was located 5.4 cm above the upper limit of vertical travel of the source. The absorber was placed 4.7 cm from the counter face. A single-channel pulse-height analyzer selected the 14.4-kev gamma ray photopeak. The outputs of this analyzer and the 1000 cps clock were switched between two pairs of scalers so as to record counting rate and velocity separately for both directions of source motion.

Figure 1 shows the counting rate (in arbitrary units) as a function of source velocity relative to the Fe_2O_3 absorber for the stainless steel source. Absorption of the "unsplit" emission line at each of the six possible absorption energies of Fe^{57} in Fe_2O_3 is evident. The lack of symmetry of the absorption pattern about zero velocity shows immediately that one is not dealing simply with a magnetic hyperfine splitting pattern.

The velocities at which absorption peaks occur are given in Table I. Experimental values have been corrected by -2.5% to take account of the effect of geometry on our velocity scale. These absorption line velocities may be fitted precisely in terms of an energy level diagram as shown in Fig. 2. It is necessary to introduce an energy shift $\Delta E = \Delta E_1 + \Delta E_2$ between the center of gravity of the absorption lines of Fe^{57} in Fe_2O_3 and the emission line of Fe^{57} in stainless steel. In addition, an energy shift ϵ, of positive or negative sign, on the individual m states of the excited level is required. This is interpreted as being caused by a quadrupole interaction when Fe^{57} is bound in Fe_2O_3. A least-squares fit to our data yields the following splitting parameters (in "velocity units") for the two Fe^{57} nuclear states when Fe^{57} is bound in Fe_2O_3:

$$g_0' = 0.611 \pm 0.005 \text{ cm/sec},$$

$$g_1' = 0.345 \pm 0.003 \text{ cm/sec},$$

$$\Delta E = 0.047 \pm 0.003 \text{ cm/sec},$$

$$\epsilon = 0.012 \pm 0.003 \text{ cm/sec}.$$

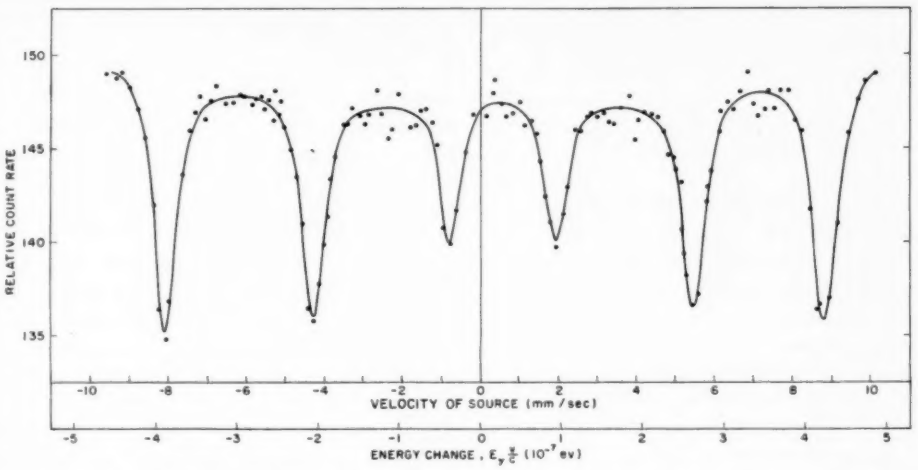

FIG. 1. The absorption by Fe^{57} bound in Fe_2O_3 of the 14.4-kev gamma ray emitted in the decay of Fe^{57m} bound in stainless steel as a function of relative source-absorber velocity. Positive velocity indicates a motion of source toward absorber.

These data indicate that the ratio g_0'/g_1' is 1.77 ± 0.02, in excellent agreement with the results of Hanna et al.,[4] and that the internal magnetic field in antiferromagnetic Fe_2O_3 is larger than in ferromagnetic iron by the factor 1.547 ± 0.022. If we take 3.33×10^5 oersteds as the value of the internal field[4] in Fe, we find a value of 5.15×10^5 oersteds at the Fe nucleus in Fe_2O_3. It is worth noting that the absorption peaks in Fig. 1 (reading from left to right) should have "thin absorber" intensity ratios of 3:2:1:1:2:3. However, the relative intensities found for the absorption peaks clearly exhibit evidence of saturation due to the rather large effective thickness of the enriched Fe_2O_3 absorber. Nevertheless, the experimental intensity ratios are such that an inverted hyperfine pattern is required for the 3/2- state, providing confirmation for this fact as previously reported.[4]

Let us now consider the effect of quadrupole

Table I. Velocities of maximum absorption (corrected for geometry) for a stainless steel source and Fe_2O_3 absorber. The negative signs indicate motions of source away from absorber.

Absorption line	Relative source-absorber velocity (cm/sec)
1/2+ → 3/2+	-0.789
1/2+ → 1/2+	-0.417
1/2+ → 1/2-	-0.076
1/2- → 1/2+	+0.192
1/2- → 1/2-	+0.536
1/2- → 3/2-	+0.859

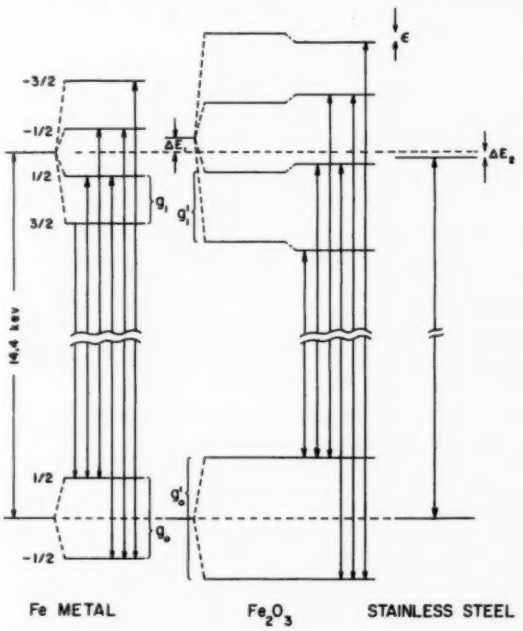

FIG. 2. Schematic representation of the ground and 14.4-kev excited states of Fe^{57} bound in ordinary iron, Fe_2O_3, and stainless steel. This diagram illustrates the details of magnetic hyperfine splitting, quadrupole interaction, and energy shifts due to chemical binding effects.

interaction. For a pure quadrupole spectrum, in the case of axially symmetric field gradients, the shift ϵ for the substates of a spin I state is given by[6]

$$\epsilon = \frac{e^2qQ}{4I(2I-1)}[3m^2 - I(I+1)],$$

where q denotes $(1/e)(\partial^2 V/\partial z^2)$. All other symbols have the conventional meanings. When Zeeman splitting is combined with quadrupole interaction, the precise energy shifts for the individual m states due to quadrupole interaction depend upon the orientation of the magnetic axis relative to the axis of symmetry for the electric field gradient, and are in general not the same for all m states.[7] However, for a case such as we deal with here, where the quadrupole interaction is small compared to the magnetic interaction, the absolute magnitude of ϵ is the same for all m states of the spin 3/2 state within the accuracy of our measurements, independent of the orientation of the crystal axes. Thus, we find $\epsilon = \frac{1}{4}|e^2qQ| = 0.012 \pm 0.003$ cm/sec (5.75×10^{-9} ev). Since our measurements indicate that the shift of magnitude ϵ is negative for the $m = \pm 3/2$ substates and positive for the $m = \pm 1/2$ substates, the product e^2qQ is negative. The absolute value is suggestive of a small quadrupole moment for Fe^{57m} although this statement cannot be amplified further without knowledge of electric field gradients in Fe_2O_3.

In our earlier absorption measurements on Fe_2O_3 using the emission lines of Fe^{57} bound in ordinary iron, we find an energy shift upward (see Fig. 2) of $\Delta E_1 \cong 0.04$ cm/sec. Within the accuracy of our measurements, the splitting parameters g_0', g_1', and ϵ remain the same. A separate measurement of the absorption by Fe^{57} bound in stainless steel of the emission lines of Fe^{57} in ordinary iron confirms the implied existence of an energy shift downward (see Fig. 2) of $\Delta E_2 = 0.01$ cm/sec in this case.

The existence of an energy shift $\Delta E = (2.26 \pm 0.15) \times 10^{-8}$ ev between stainless steel and Fe_2O_3 has been definitely established. We will now discuss the ways in which differences in chemical environment may produce such a shift. (1) Source and absorber will in general have different Debye temperatures. Since the nucleus in its excited state has a slightly greater rest mass than when it is in its ground state, the energy difference between the nuclear states will decrease from its value for an "unbound" system by different amounts because of the difference in the zero-point energies. This reduction in energy is larger for the substance having the higher Debye temperature. (2) In addition, when a lattice is at a finite temperature, the energy difference between the nuclear states is reduced further by virtue of the previously reported temperature effect.[8] Since the magnitude of this reduction depends upon the integral of the specific heat of the lattice up to the temperature of the substance, it will be larger for the substance having the lower Debye temperature. We expect this temperature effect to be small, although it should be noted that it acts to reduce the energy shift between different substances which results from zero-point energy differences. The sign and magnitude of the observed shift between stainless steel and Fe_2O_3 would require that Fe_2O_3 have a considerably lower Debye temperature than stainless steel or ordinary iron. Were this the case, the Debye-Waller factor would materially depress the recoil-free resonant yield in Fe_2O_3, a result which is not indicated by the data. (3) When chemical environment is altered, a nuclear isotope shift may result. This effect has its origin in the change in the electronic wave functions over the region of space occupied by the nucleus. s electrons may be expected to contribute most to this effect. Since s electrons are in effect removed in going from Fe in metal to Fe in Fe_2O_3, a smaller charge radius for Fe^{57m} than for Fe^{57} in its ground state would produce a shift in the observed direction. The direction of the observed energy shift ΔE requires the presence of the nuclear isotope shift, since the zero-point energy shift is in the opposite direction. This nuclear isotope shift is similar to the isomeric isotope shift[9] observed in the optical spectrum of Hg^{197}.

We wish to emphasize two additional points about recoil-free emission and absorption experiments when source and absorber are chemically different. First, the existence of energy shifts introduces asymmetries into the absorption pattern which make it essential not to combine data taken at equal velocities of opposite sense. Second, any recoil-free absorption at zero velocity is accidental. In general, therefore, a search for this effect by comparing absorption at zero velocity with absorption at a large relative velocity between source and absorber will not yield significant results unless source and absorber are identical chemically. It

is amusing to note that in the absorption by Fe_2O_3 of the emission lines of Fe^{57} in ordinary iron, two such accidental coincidences do indeed occur to give substantial absorption at zero velocity.

We wish to thank many of our colleagues, particularly M. Goldhaber and J. Weneser, for interesting discussions, and G. K. Wertheim for providing us with a sample of the particular stainless steel used in his measurements.[5]

*Work done under the auspices of the U. S. Atomic Energy Commission.

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STRUCTURE OF NUCLEAR MATTER

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(Received March 17, 1960)

It is usually assumed that the Hartree (or Hartree-Fock) ground state of a Fermi gas is the familiar sphere of occupied momentum states. For a noninteracting gas such is, of course, the ground state; and for an interacting gas, this state is, to be sure, an exact solution of the Hartree-Fock equations. But to regard it as the lowest single-particle state has always been an unproved assumption, and indeed a false one. Explicit Hartree states of lower energy will be displayed below for a Fermi gas with attractive interactions. This problem is of practical interest with regard to nuclear matter, and will be discussed within that context. It will be shown that there are large static density waves in nuclear matter, so that (to an adequate approximation) the total nucleon density has the following spatial variation:

$$\rho=\rho_0(1+\gamma\cos qx)(1+\gamma\cos qy)(1+\gamma\cos qz). \quad (1)$$

In another paper it will be shown that a low-energy Hartree-Fock state of a Fermi gas with repulsive interactions will have large spin density waves. It should not be necessary to emphasize that these new states have not been proved to be the ground state in the single-particle approximation, although they do have considerably lower energy than the "normal ground state." However, it seems clear that any attempt to improve upon the single-particle approximation, such as the many-body techniques developed in recent years, should begin with the lowest Hartree-Fock state available.

It is not our purpose here to present a general argument, applicable to all conceivable interactions, but rather to illustrate the physical structure of the new low-energy states within the framework of a simple model. We shall treat first a one-dimensional problem, and afterwards a three-dimensional nucleon gas. Consider a large number N of spinless Fermi particles in a one-dimensional box of length L. The kinetic energy operator will be the usual one. We shall assume the total interaction energy of the gas to be

$$V=-(\beta L/2N)\int_0^L \rho^2 dx, \quad (2)$$

where $\rho(x)$ is the total particle density and β a constant. If the difference between N and N-1 is neglected, the Hartree potential $U(x)$ experienced by any one of the particles is

$$U(x)=-(\beta L/N)\rho(x). \quad (3)$$

[The results to be derived are by no means limited to the particular interaction (2). For example, any power of ρ may be used in the inte-

grand of (2). However, the requirement that the equilibrium density be finite restricts that power to values between 1 and 3.] The single-particle quantum states will be indexed according to the wave vectors allowed by periodic boundary conditions. The usual momentum states will have energy $2\omega_k E_F$, where

$$\omega_k = \tfrac{1}{2}(k/k_0)^2, \tag{4}$$

and E_F is the Fermi energy, $\hbar^2 k_0^2/2m$, with $k_0 = \pi N/L$, the wave number at the Fermi surface.

Consider now the following single-particle wave functions:

$$\phi_k = \left[ge^{ikx} + (E_k - \omega_k)e^{i(k-q)x}\right] \Big/ L^{1/2}\left[g^2 + (E_k - \omega_k)^2\right]^{1/2}, \tag{5a}$$

for $0 \leq k \leq k_0$; and

$$\phi_k = \left[ge^{ikx} + (E_k - \omega_k)e^{i(k+q)x}\right] \Big/ L^{1/2}\left[g^2 + (E_k - \omega_k)^2\right]^{1/2}, \tag{5b}$$

for $-k_0 \leq k \leq 0$. The energy E_k, appearing parametrically, is

$$E_k = \tfrac{1}{2}(\omega_k + \omega_{k-q}) - \left[\tfrac{1}{4}(\omega_k - \omega_{k-q})^2 + g^2\right]^{1/2}, \tag{6}$$

for positive k, and the corresponding expression with $q = 2k_0$ replaced by $-q$ for negative k. The foregoing wave functions and energies are the solutions of a single particle in a potential,

$$U = 4gE_F \cos 2k_0 x, \tag{7}$$

provided admixture of states with $|k| > 2k_0$ is ignored. (These are the wave functions and energies which arise in the weak-binding theory of energy bands. The energy gap at $\pm k_0$ is $4gE_F$.) The qualitative feature of the wave functions (5) is that their square magnitudes have an oscillating component out of phase with the potential (7), and give rise to a density wave which can in turn yield (7) as the Hartree potential of the attractive interactions. States corresponding to $|k| > k_0$ may be defined similarly, but should not be occupied, since their square magnitudes oscillate in phase with (7). This is the reason why the wave number of the density wave should equal the diameter, $2k_0$, of the Fermi surface. (Only those states below the energy gap should be filled.) It can easily be proved that this choice leads to the lowest total energy. But we must first show that a Fermi sea filled with the states (5) has lower energy than one filled with plane waves.

The kinetic energy T of a Fermi gas filled with the distorted states (5) can be (tediously) evaluated in a straightforward manner:

$$T = \tfrac{1}{3}NE_F[1 + 3F(g)], \tag{8}$$

where

$$F(g) = 1 - (1+g^2)^{1/2} + g^2 \ln\left[g^{-1} + (1+g^{-2})^{1/2}\right]. \tag{9}$$

The first term of (8) is the kinetic energy of a normal gas, whereas the second is the increase arising from the assumed distortion of the N-particle (simple product) wave function. The density of the gas can also be calculated directly, and is

$$\rho = (N/L)[1 - G(g)\cos qx], \tag{10}$$

where

$$G(g) = g \ln\left[g^{-1} + (1+g^{-2})^{1/2}\right]. \tag{11}$$

As surmised, the state has a static density wave. This result together with (2) determines the interaction energy:

$$V = -\tfrac{1}{2}\beta N(1 + \tfrac{1}{2}G^2). \tag{12}$$

If F and G are expanded in powers of g, it can be made obvious that V will always decrease faster than the increase of T near $g = 0$. Actually, the sum of (8) and (12) can be minimized without approximation. The optimum g is

$$g = 1/\sinh(4E_F/\beta). \tag{13}$$

The resulting change in total energy relative to the normal state is

$$\Delta W = -NE_F[\coth(4E_F/\beta) - 1]. \tag{14}$$

This result is negative definite. It is also of interest to note that if (10) and (13) are inserted into (3), one obtains a constant term plus the oscillating potential (7), the coefficient of the latter being in agreement with (13). Consequently, the N-particle wave function is a self-consistent solution in the Hartree sense (states with $|k| > 2k_0$ being ignored). A further observation is that within the field of variation employed here, the "normal ground state" is an energy maximum.

The generalization to a three-dimensional gas is almost trivial. In order to optimize the effect of density waves, the occupied (unperturbed) states should fill a cube in momentum space. Three density waves will result, according to (1), and there will be a large energy gap around the surface of the Fermi cube. (The wave number q of the waves must equal the edge length of

the cube.) The single-particle wave functions are merely the products of three factors, each having the form (5). To a first approximation each of the density waves can be treated independently, and the analysis is identical to that derived above. However, employment of an interaction term such as (2) does not lead to an equilibrium density for nuclear matter.

The energetics of nuclear density waves for a model which exhibits nuclear saturation may be explored easily with the method of Karplus and Watson.[1] According to their treatment, the binding energy per nucleon of the normal state is (in Mev)

$$B = -(23/\eta^2) + (V_0/2\eta^3)[1-(3\alpha/10\eta^2)], \qquad (15)$$

where η is a radius parameter, $\eta^3 = \rho_0/\rho$, ρ_0 being the observed nuclear density. The parameters V_0 and α may be estimated from nucleon scattering experiments or, for our purposes, may be chosen to yield the observed binding energy (~16 Mev) and density ($\eta \sim 1$) of nuclear matter. The first term of (15) is the kinetic energy per nucleon, and the second is the potential. The negative term in square brackets represents the decrease in attractive interaction associated with the mean square relative momentum P of the nucleons:

$$P \sim \sum_{i,j} |\vec{k}_i - \vec{k}_j|^2. \qquad (16)$$

[The physical origin of this term is of course the penetration of the repulsive core by nucleons of high relative momenta. The potential terms of (15) and (2) are equivalent if β is allowed to have this dependence. Nuclear saturation is thereby guaranteed.] The appropriate modification of (15) to include the effects of three perpendicular density waves is

$$B' = -\frac{23}{\eta^2}(1+3F) + \frac{V_0}{2\eta^3}(1+\tfrac{1}{2}G^2)^3\left[1-\frac{3\alpha}{10\eta^2}(1+\tfrac{3}{2}F)\right], \qquad (17)$$

where $F(g)$ and $G(g)$ are as defined by (9) and (11). For excitations near the Fermi surface, the fractional increase in P is one half the fractional increase in kinetic energy. Equation (17) should first be minimized with respect to g. If only linear terms in F and G^2 are employed in finding the optimum g, the result is analytic:

$$g = 1/\sinh\theta, \qquad (18)$$

where

$$\theta = (2760\eta^3 + 9\alpha V_0)/V_0(30\eta^2 - 9\alpha). \qquad (19)$$

We have computed B and B' vs η for various sets of V_0 and α. The set, $V_0 \sim 120$ Mev, $\alpha \sim 1.4$, yields the observed density and binding energy for B'. The extra binding energy $(B'_{\max} - B_{\max})$ attributable to the density waves is 4 Mev per nucleon. The energy gap, $g\hbar^2q^2/2m$, at the Fermi surface is 30 Mev. The amplitude γ of each density wave is 0.6. These quantitative results are relatively insensitive to the values of V_0 and α. One may conclude that density waves in nuclear matter are of great importance.

The foregoing arguments have the force of the variational theorem. The density waves are so large that the variational solution should be improved appreciably by allowing states with wave vector components greater than q to be mixed into the single-particle wave functions. Also, the optimum value of g will be different from the crude estimate (18). Such refinements can only enhance the importance of density waves and increase their contribution to the binding energy. In writing (17), we have neglected the 8% increase in kinetic energy required to fill a cube rather than a sphere in momentum space. Most of this increase arises from filling the corners of the cube. It seems likely that the cube will be truncated by four smaller density waves parallel to the [111] type directions. The effect on the three major density waves should be small, but a major fraction of the 8% increase in kinetic energy can thereby be eliminated.

In the limit of weak attractive interactions, the Fermi surface will become a many-faced polyhedron, each pair of opposite faces arising from a density wave. All single-particle states near the Fermi surface will be essentially 100% perturbed compared to the normal state. It follows that the low-energy states proposed in this paper are not related to the normal state through a perturbation expansion in the interaction constant. The stability of static density waves arises from the fact that the greater binding caused by density fluctuations is achieved with less expenditure of kinetic energy, when compared with a mere uniform compression of the normal state. This observation points out the inadequacy of the Fermi-Thomas approximation for such considerations.

Glassgold, Heckrotte, and Watson[2] found that nuclear matter is unstable with respect to compressional collective modes. They assumed, of

course, that the parentage of the true ground state is the normal free particle state. The interpretation of their result is now clear. Compressional modes relative to a highly excited "ground state" are unstable if they have sufficient admixture of low-energy states, similar to those considered here.

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REGENERATION AND MASS DIFFERENCE OF NEUTRAL K MESONS*

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A very significant feature of the Gell-Mann–Pais particle mixture theory[1,2] is the regeneration of the $K1$ from the $K2$ neutral meson. We examine the three possible types of regeneration and give the results of an experiment that exhibits the expected transformations as demanded by the theory. The experiment also allows an estimate of the difference between the masses of $K1$ and $K2$.

One of the three types of regeneration has been described previously[3]: A plate inserted into a parallel beam of $K2$ particles produces a parallel beam of $K1$ particles. This phenomenon, which we will henceforth call transmission-regeneration, is in striking contrast with other known processes whereby a particle transforms into another one: a parallel beam of charged pions obviously cannot produce a parallel beam of neutral pions by interacting with a plate.

Here we point out another process that typically follows from the theory, namely the regeneration by diffraction. Because the $\overline{K}^0$ and the K^0 waves are diffracted by a nucleus with different amplitudes, the diffracted wave contains $K1$ as well as $K2$ particles. Thus $K1$ mesons are regenerated by a nucleus with a typical diffraction angular distribution.

Regeneration of $K1$ can also occur by interaction of $K2$ with single nucleons. The angular distribution of this nucleon-regeneration is broad, not essentially different from that obtained in K-nucleon scattering, and therefore it is not a crucial consequence of the particle mixture theory.

All three of these components will emerge from a plate traversed by a parallel beam of $K2$'s. The angular distribution should permit one to recognize each component separately.

Case[4] and Good[5] have shown that the intensity of the transmitted component is a very sensitive function of the mean life τ_1 of the $K1$ and of the difference δm between the masses of $K1$ and $K2$. The mass difference appears in the final expression because of the phase difference it introduces between the $K1$ and the $K2$ waves, an effect which was first noted by Serber[6] in connection with K^0 production. Moreover, Good pointed out that the intensities of both the transmitted and "scattered" component (in the forward direction) are proportional to $|f_{21}{}^0|^2$, $f_{21}{}^0$ being the amplitude of the regenerated $K1$, at zero angle, in a $K2$-nucleus collision. Good's "scattered" component must be identified with the diffracted component described above. Thus the intensity ratio of the transmitted wave to diffracted wave is a function only of δm and τ_1. We derive here in a more concise way the expression for this ratio.

The computation of the expected transmitted and diffracted intensities can be greatly simplified by neglecting, from the start, the regeneration of $K2$ from $K1$. As the number of $K1$'s is always less than one thousandth of the number of $K2$'s, this approximation is very good. We consider then a plane wave of $K2$ particles, of wavelength λ, crossing our plate, which contains N nuclei per cubic centimeter. If each nucleus produces $K1$'s with a forward amplitude $f_{21}{}^0$, an infinitesimal thickness dx of the plate at depth x

(x in the direction of the incoming $K2$ beam; $x=0$ and $x=L$ denote the limits of the plate) produces a $K1$ wave amplitude $iN\lambda f_{21}{}^0 dx$ which arrives at the end of the plate with the amplitude

$$da_1 = iN\lambda f_{21}{}^0 dx \exp\left(-ik_2 x - ik_1(L-x) - \frac{L-x}{2v\gamma\tau_1} - \frac{L}{2u}\right).$$

Notice that the $K2$ wave has traveled to depth x before producing the $K1$ wave, which then travels from x to L; u is the collision mean free path, which is the same for $K1$ and $K2$, because both particles are a half-and-half mixture of K^0 and $\overline{K}{}^0$; v is the velocity of the particles; γ is the Lorentz factor; $\hbar k_1$ and $\hbar k_2$ are the momenta. Let us call $\Lambda = v\gamma\tau_1$ the decay mean free path of the $K1$'s and introduce the dimensionless quantities $l = L/\Lambda$ and $\delta = (m_2 - m_1)c^2/(\hbar/c_1)$. By integration with respect to x we obtain, for the transmitted intensity,

$$|a|^2 = \frac{4|f_{21}{}^0|^2 N^2\Lambda^2\lambda^2}{1+4\delta^2}\left|e^{-i\delta l} - e^{-l/2}\right|^2 e^{-L/u}. \quad (1)$$

On the other hand, the nuclei, incoherently from each other, regenerate $K1$'s by diffraction with a differential cross section $d\sigma_{21}/d\omega = |f_{21}|^2$. The number of diffraction-regenerated $K1$'s in the infinitesimal thickness dx at x, in the forward direction, surviving through the thickness $L-x$ is

$$d\left(\frac{dn_1}{d\omega}\right)^0 = |f_{21}{}^0|^2 N \exp\left(-\frac{L-x}{v\gamma\tau_1} - \frac{L}{u}\right)dx,$$

which is integrated to give

$$\left(\frac{dn_1}{d\omega}\right)^0 = |f_{21}{}^0|^2 N\Lambda(1-e^{-l})e^{-L/u}. \quad (2)$$

The ratio between (1) and (2) is

$$R = 4N\Lambda\lambda^2\left|e^{-i\delta l} - e^{-l/2}\right|^2/[(1-e^{-l})(1+4\delta^2)]. \quad (3)$$

To observe these regeneration processes, we have inserted a plate in the Berkeley 30-inch propane chamber. The chamber was placed in a beam of $K3$ particles which traversed the instrument lengthwise and perpendicular to the plate. The experimental setup will be described in more detail in a later article. Here we give only a brief description.

A beam of 1.1-Bev/c negative pions impinged on a five-foot hydrogen target. The 670-Mev/c K^0 produced in the target travelled a distance of 22.5 feet before arriving at the 30-inch propane chamber, so that approximately one $K2$ crossed the chamber per 10^{11} protons in the Bevatron beam. About 200 000 pictures were taken, half of them with a 1.5-inch iron plate to enhance the diffracted wave relative to the transmitted wave, and the other half with a 6-inch iron plate which yields an intense transmitted wave.

We limited the analysis to those two-prong events in which the positive prong could be recognized to be a meson on the basis of ionization and momentum. We also required that the decay occur within two mean lives from the plate and that the primary momentum be equal, within the errors, to the beam momentum. The $Q(\pi,\pi)$ distribution of these selected events shows a marked peak around the expected value of 220 Mev, which fact proves the regeneration of $K1$. As a further selection, we keep only those events for which Q differs by no more than 1.4 standard deviations from the peak value.

By measuring the vector momenta of the two prongs, we determine the angle θ between the $K1$ and the incident $K2$ beams, within an error of about 2 degrees.

The angular distributions are shown in Fig. 1. The diffraction curve has been computed with an optical model method[7] using the known cross sections for K^+ and K^- with protons and nuclei.[8] The curve is quite close to the black-sphere distribution. Figure 1 clearly shows a diffraction component and a superimposed transmission peak. Most of the transmission peak is confined to angles smaller than 2.5 degrees ($\cos\theta > 0.999$), which is just what we expect from an infinitely narrow peak measured with our errors. The mere presence of such a large transmission peak is a proof that the mass difference is smaller than, say, $5\hbar/\tau_1$.

Referring to the data for the 6-inch plate, 29 events occur in the interval for $\cos\theta$ between 0.998 (3.5°) and 1, which should contain the total number T of the transmitted $K1$'s and a part D of the diffracted ones. Knowing the diffraction angular distribution we can compute D from the 31 particles in the 0.980 to 0.998 interval, assuming that that region contains only diffracted $K1$'s. A nucleonic background will actually be present in this interval, which we ignore for the moment. We thus obtain $D=12$, which gives $T=17$. For comparison with formula (3), we note that $(dn_1/d\omega) = 1.18(D/\omega)$, where the factor 1.18 is equal to the ratio of the intensity at zero degrees to the average intensity within ω, ω being the solid angle in the peak interval, that is, $2\pi \times 0.002$. It is convenient to compare $T/1.18D$

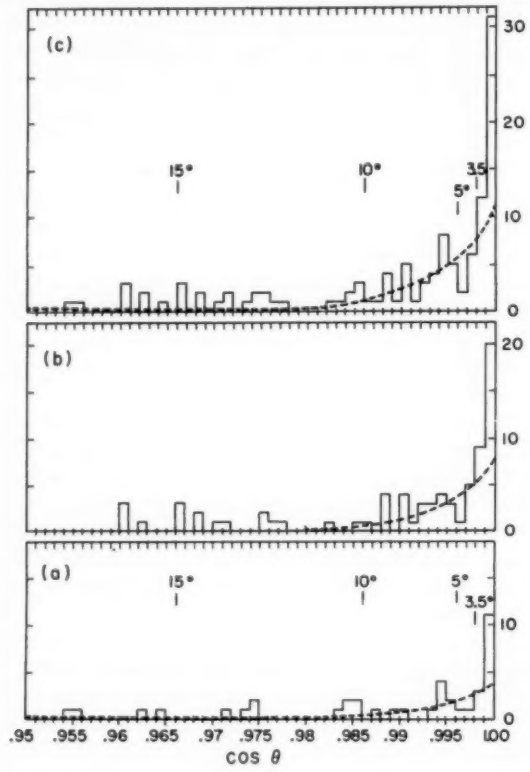

FIG. 1. Histograms of number of $K1$ decay events per 0.001 interval of $\cos\theta$ (θ is the angle between the direction of the primary $K2$ beam and the regenerated $K1$). (a) Data for the 1.5-inch plate; (b) data for the 6-inch plate; (c) combined data for the two plates. The curves are diffraction angular distributions normalized in the 0.980 to 0.998 interval for $\cos\theta$.

with R/ω, which is plotted in Fig. 2 versus the mass difference, in units of $\hbar/\tau_1$. We obtain $T/1.18D = 1.2 \pm 0.53$, which gives $\delta m = 0.85^{+0.4}_{-0.25}$. With a probability of 95%, $\delta m < 1.4$.

In the same way, we find from the thin plate alone $\delta m = 0$ ($\delta m < 4.5$ with 95% probability) and from the combination of thin and thick plate $\delta m = 0.85^{+0.3}_{-0.25}$ ($\delta m < 1.5$ with 95% probability).

If we correct for nucleonic or any other background, we would obtain a larger value for R, hence a smaller value for δm. For instance, assuming a uniform background from $\cos\theta = 0.96$ to $\cos\theta = 1$, the compounded data for both plates yield $\delta m = 0$ ($\delta m < 1.1$ with 95% probability). In view of a remark by Okun' and Pontecorvo,[9] this result indicates that decay rates for $\Delta S = 2$ are 10^5 times slower than for $\Delta S = 1$.

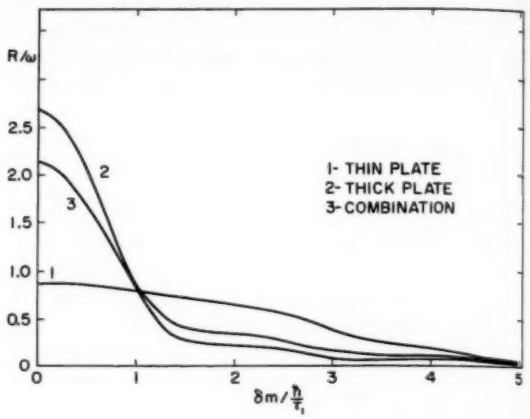

FIG. 2. Calculated intensity of the forward regenerated $K1$'s versus the $K1$-$K2$ mass difference. The ordinate R/ω is the ratio of the transmission-regenerated $K1$'s to the diffraction-regenerated $K1$'s in the interval $\cos\theta > 0.998$.

We are grateful to many people who generously contributed to this experiment, particularly Edward J. Lofgren, Myron L. Good, Richard L. Lander, Robert E. Lanou, Marian N. Whitehead, Roy Kerth, and Frank T. Solmitz. The pictures have been scanned by J. Peter Berge, Karl Brunstein, Layton Linch, Mrs. Glennette Anneson, Mrs. Rokalana Gamow, and Mrs. Ottilie Oldenbusch.

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

† On leave of absence from Ecole Polytechnique, Paris and Marine Nationale Francaise.

‡ Now at Brookhaven National Laboratory, Upton, New York.

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COSMIC RADIATION INTENSITY DECREASES OBSERVED AT THE EARTH AND IN THE NEARBY PLANETARY MEDIUM*

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(Received March 29, 1960)

This note reports preliminary experiments which prove that existing theories for the modulation mechanism responsible for rapid decreases of primary cosmic-ray intensity cannot invoke the presence of the earth or its magnetic field.

Forbush first noted sudden intensity decreases in ionization chambers located deep within the atmosphere which followed in the order of 20 to 40 hours some large solar flares.[1] These intensity decreases arose from changes in the secondary radiation produced by primary cosmic rays and were frequently accompanied by substantial geomagnetic field disturbances. It was later shown experimentally that this kind of world-wide decrease was a property of the primary cosmic radiation observed at the earth, and arose neither as a consequence of the geomagnetic storm nor by phenomena which might change temporarily the magnetic cutoff rigidities, such as ring currents around the earth.[2] The changes in the primary spectrum during times of drepressed intensity—namely, a larger decrease for low magnetic rigidity particles than for high rigidity particles—suggested that the modulation was due to interplanetary magnetic fields in the vicinity of the earth.[2] For a typical event, the high rate for reduction of prevailing cosmic radiation intensity at the earth also placed bounds on the requirements for magnetic field intensity and scale size of suitable modulating electromagnetic fields, i.e., intense fields of large scale size, or vice versa.

Several hypotheses have been advanced for this solar-produced modulation of galactic cosmic radiation. On the one hand are models which may be described roughly as heliocentric; namely, where the cosmic radiation intensity in a substantial volume of the inner solar system is reduced by either disordered,[3] or ordered magnetic fields[4,5] of solar origin independent of the presence of the earth and its geomagnetic field. On the other hand are models which depend upon the solid earth and its permanent magnetic field for creating the decrease of cosmic-ray intensity; namely, geocentric models.[6] To decide between these two classes of hypotheses, we ask the question: How far into the interplanetary medium is the full decrease of galactic cosmic-ray intensity observed during a Forbush decrease? Is the pre-existent radiation intensity found at distances beyond which the geomagnetic field could be invoked to account for the Forbush decrease, or is the intensity also reduced in the nearby interplanetary medium as it is at the earth? For all existing models which invoke the presence of the geomagnetic field, a suitable limit for the effective extent of the modulating region is 6 to 10 R_e, where R_e is the radius of the earth.

To answer these questions, we have performed a direct experiment using a cosmic-ray detector carried by the Explorer VI satellite launched August 7, 1959 in an orbit which extended beyond the region of trapped Van Allen radiation and reached a range in excess of $7.5R_e$. In addition to the satellite observations, we measured simultaneously the changes in the nucleonic component at the earth over a wide range of geomagnetic cutoff rigidities. For this purpose we used neutron intensity monitors extending from the geomagnetic equator to high latitudes.

The cosmic-ray detector in the satellite was a triple-coincidence, proportional counter system which measured protons in excess of 75-Mev energy, or electrons in excess of 13-Mev energy. The triple coincidence detector does not respond to bremsstrahlung, such as from electrons trapped in the geomagnetic field. The satellite orbit was an ellipse of apogee approximately 48 800 km and perigee approximately 6600 km, with

its orbital plane inclined by about 46° to the geographic equator. A description of the instrument, its performance, and the satellite and its motions will appear elsewhere.[7] Most of the data recorded at the earth during the two months when the telemetry was operative covered the first four weeks after the launching. During this time, one outstanding sharp decrease of cosmic-ray intensity and two smaller decreases were observed in the neutron intensity monitors at the earth. The results we report here are for this period. Only satellite data in the range 35 000 to 48 800 km were used for analysis. The continuous record of the neutron intensity monitor at Climax, Colorado (located at a geomagnetic rigidity cutoff of ~2.5 Bv) was chosen from among the network detectors to represent the changes at the earth. Only neutron intensity data for time intervals which overlapped the satellite triple-coincidence data were used as shown in Fig. 1. The statistical errors of the satellite data are less than 1% for all points used in Fig. 1. All neutron data have smaller errors. From Fig. 1 the over-all correlation between the detectors at 35 000 to 48 000 km and at the earth's surface is obvious. Only the large decrease on August 20 provides data of sufficient accuracy to compute the ratio of the

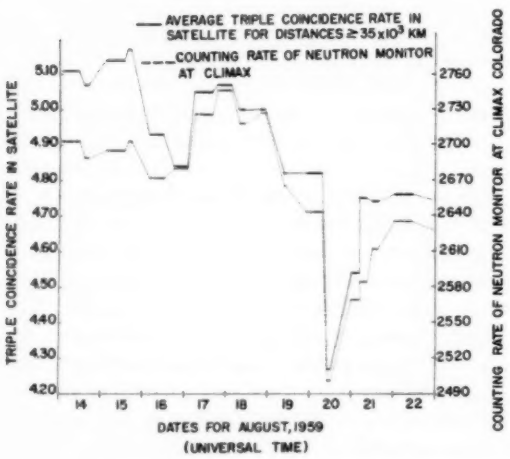

FIG. 1. Cosmic-radiation intensity as a function of time showing on August 20, 1959 a Forbush-type intensity decrease of ~15% within five hours. The nucleonic component monitor provides a measure of primary intensity changes at the earth. The light-dashed and solid lines are used to connect the periods for which satellite data are available.

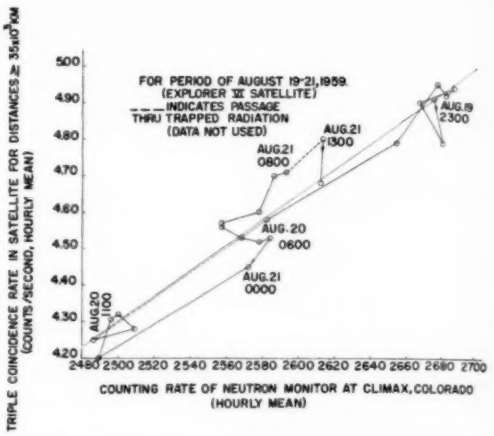

FIG. 2. The relative changes in cosmic-ray intensity at the satellite and at the earth for the sharp intensity decrease of August 20 shown in Fig. 1. The correlation coefficient is 0.96.

relative change of intensity at the satellite and at the earth as shown in Fig. 2. The ratio is:

$$\frac{\text{Relative change in satellite detector}}{\text{Rel. change in nucleonic component detector}} = 1.9.$$

The correlation coefficient for the two sets of data in Fig. 2 is 0.96. We now compare the amplitude of the Forbush decrease at great distances from the earth with the amplitude in the primary radiation at the top of the atmosphere. Therefore, we have to relate the intensity changes of secondary neutrons as observed at Climax to the changes of primary intensity outside the atmosphere above Climax. For this purpose we may use neutron intensity measurements at balloon altitudes which have been carried out for typical Forbush decreases over a number of years by our laboratory. For the present phase of the solar cycle, and cosmic-ray spectrum, this factor is approximately 1.8. Therefore, within an experimental error of approximately 10%, the magnitude of the sharp decrease of primary cosmic radiation was the same at the earth as it was out to distances as great as $7.5R_e$. Thus the decrease of August 20, 1959 means that more than 15% of all the cosmic-ray particles were removed within 5 hours by the modulation mechanism. Using neutron intensity monitor data we have shown that this event is worldwide, and that the spectrum has undergone changes which are typical of other Forbush

decreases studied at this phase of the solar cycle.

We believe this experiment excludes existing hypotheses which invoke the presence of the earth and the geomagnetic field to account for the Forbush decreases of cosmic-ray intensity. Thus, geocentric models such as solar plasmas containing disordered magnetic fields overlying the geomagnetic field, or the roughing-up of the geomagnetic field by a solar wind do not account for our observations. On the other hand, we know that these sharp intensity decreases at the earth are the delayed consequences of large solar flare outbursts on the visible side of the sun. Forbush decreases are observed at the earth only for restricted angular positions of the solar events relative to the position of the earth. Therefore, the modulation region may be confined to a volume of space surrounding the Sun-Earth line. This restricted heliocentric system of fields is different, at least in scale size, from the mechanism required for the modulation of cosmic radiation intensity over the 11-year solar activity cycle.[8] We do not here go into further details of suitable models.

The space probe Pioneer V (launched March 11, 1960) carrying equipment identical with the Explorer VI satellite may soon provide further tests of the scale size for the Forbush intensity decrease at distances $\gg 8R_e$.

Figure 1 also shows that the radiation measured by the satellite undergoes a stepwise intensity decrease following each of the two decreases which is not apparent in the relativistic energy range of the spectrum as represented by the neutron intensity monitors on the earth. This may indicate the persistent exclusion of some low-energy particles by the mechanism producing the Forbush decreases and is being investigated.

The authors wish to thank especially Leonard Petraitis, Robert Takaki, Joseph Jewzeski, James Lamport, and Lucien Biberman of the University's Laboratories for Applied Sciences for their contributions to the engineering of our apparatus in Explorer VI. The Advance Research Project Agency provided funds for the initial development of the instruments.

We wish also to express our thanks for the over-all payload engineering and coordination provided by Space Technology Laboratories, especially Dr. Adolph Thiel and Dr. John Lindner. The constant interest of Dr. John Lindsay of the National Aeronautics and Space Administration was deeply appreciated. Mr. R. Tjonaman and staff of the Enrico Fermi Institute Cosmic-Ray Group maintained a continuous watch for unusual solar and cosmic-ray events which made possible the increased data acquisition from the satellite during August.

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*This work was supported in part by the National Aeronautics and Space Administration and in part by the U. S. Air Force Office of Scientific Research.

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ANALYSIS OF THE EXPERIMENTAL τ'^{+} DECAY SPECTRUM AS A TEST OF THE $\Delta T = \frac{1}{2}$ RULE*

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A further test of the isotopic spin selection rule $\Delta T = \frac{1}{2}$ has been recently proposed by Weinberg.[1] For the form of the energy spectrum of the odd pion in τ or τ' decay, Weinberg writes:

$$p\left(\frac{T_3}{m_K}\right) \sim \frac{3m_K}{Q}\left(\frac{1}{1+aQ/3m_K}\right)\left(1+\frac{aT_3}{m_K}\right) x_{\max}\left(\frac{T_3}{m_K}\right), \quad (1)$$

where T_3 is the kinetic energy of the odd pion, Q is the total kinetic energy of the three pions, m_K is the mass of the K meson, and $x_{\max}$ is the maximum value of the x coordinate of the Dalitz plot. Weinberg shows that if the decay follows the $\Delta T = \frac{1}{2}$ rule, then the value of a for the τ' decay is uniquely determined by the value of a for the τ decay. From a recent study of 959 τ events[2] Weinberg obtains $a_\tau \approx 4.9$ and predicts $a_{\tau'} = -2a_\tau \approx 9.8$.

However, this value for $a_{\tau'}$ is not consistent with taking only a linear term in T_3 in Eq. (1), as p would become negative for large T_3. Hence, the smallest value of $a_{\tau'}$ consistent with p remaining everywhere positive, $a_{\tau'} = -9.3$, has been used in this communication as the predicted value. This value is well within the errors, since the intrinsic uncertainty in Weinberg's prediction, due to neglect of quadratic terms in Eq. (1), is stated to be about 20%, while the experimental error is also about 20%.

Seventy-two τ' decays, previously reported by the Columbia Emulsions Group,[3] have been reanalyzed to measure the value of $a_{\tau'}$. The likelihood function obtained from Eq. (1), after normalization, is

$$P = \prod_{i=1}^{72} p_i = \left(\frac{K}{1+a_{\tau'}/17.76}\right)^{72} \prod_{i=1}^{72}\left(1+\frac{a_{\tau'}T_{3i}}{m_K}\right) x_{\max i}\left(\frac{T_{3i}}{m_K}\right), \quad (2)$$

where K is a constant which does not depend on $a_{\tau'}$. The relativistic boundary of the Dalitz plot was used in the normalization. A plot of the likelihood function is shown in Fig. 1. The maximum likelihood value for $a_{\tau'}$ is -7.1, as compared to the value of -9.3 predicted by Weinberg's analysis. At $a_{\tau'} = -9.3$, the likelihood function has dropped by a factor of about 6. At a value of $a_{\tau'} = 4.6$ (minus one half the predicted value of -9.3 and hence corresponding to the τ value), the likelihood function is down by a factor of about 270.

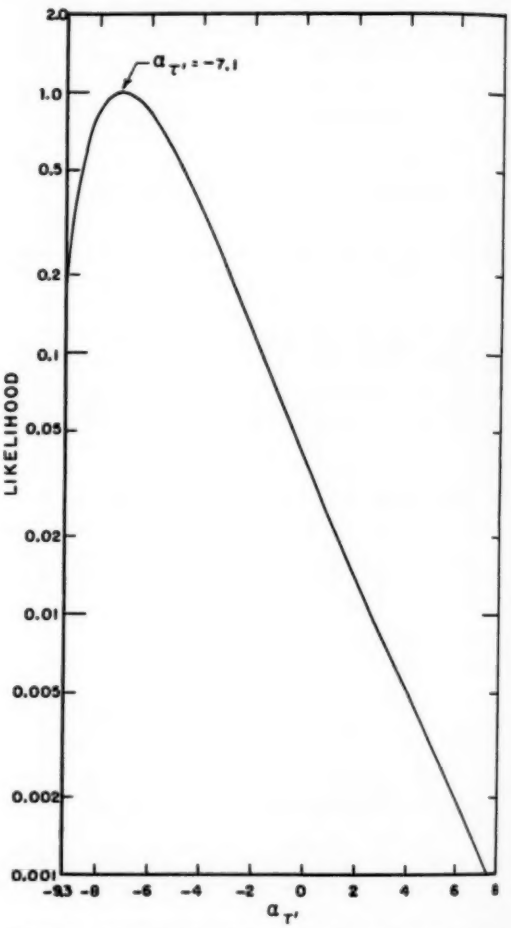

FIG. 1. The likelihood function, or the relative probability that our seventy-two events turn out the way they did, has been plotted as a function of the parameter $a_{\tau'}$.

Six-division χ^2 tests have also been carried out for various values of $a_{\tau'}$, yielding the results given in Table I. The sensitivity of the data to

Table I. Results of the χ^2 tests of the data for various assumed values of the parameter $a_{\tau'}$.

$a_{\tau'}$	χ^2	Probability	Remarks
4.6	13.6	0.017	τ value
0.0	8.8[a]	0.12	
-7.1	3.8	0.57	maximum likelihood value
-9.3	8.2	0.14	predicted value

[a]This value of χ^2 differs from that in reference 3, since here the relativistic boundary of the Dalitz plot was used, while reference 3 used the nonrelativistic approximation.

errors in measuring the kinetic energies has been tested by recalculating the maximum likelihood value of $a_{\tau'}$ using the upper and lower error limits of the most sensitive event of the data, which also happens to have large errors. This event has $T_3=49.0^{+4.3}_{-6.0}$ Mev. The maximum likelihood values so obtained are $a_{\tau'}=-6.8$ and $a_{\tau'}=-7.4$ for the upper and lower limits, respectively.

The measured value $a_{\tau'}=-7.1$ is in satisfactory agreement with the predicted value of -9.3. Hence, the present data, as tested by Weinberg's analysis, are consistent with a $\Delta T=\frac{1}{2}$ rule. It is unlikely that $a_{\tau'}$ is as large as a_{τ}, although the possibility cannot be completely excluded by the present data.

We thank Mr. T. Faith for his aid in carrying out the calculations.

*Supported in part by a grant from the National Science Foundation.

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AUGER EFFECT IN MESONIC ATOMS

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The Auger effect is important in numerous experiments dealing with the capture of negative particles by nuclei, because of the dependence of the interpretation of such experiments on the details of the atomic cascade process. It is often essential to know, in the case of a strongly interacting negative particle, in which state it undergoes interaction with the nucleus. The answer to this question depends in turn upon the transition rates out of the various states. Therefore it is necessary to have a reliable theory of the Auger transition rate. Unfortunately, although the usual perturbation theory of the Auger effect is straightforward, it has been put into doubt by the serious discrepancies found by Stearns and Stearns[1] for the x-ray yields in μ- and π-mesonic atoms. The purpose of the present note is to point out an alternative way of computing the Auger effect which makes it clear that the usual perturbation treatment is absolutely reliable. In fact, we demonstrate a quantitative relationship between the Auger rate on the one hand and the photoelectric cross section on the other. From this relationship, it is possible to calculate the Auger effect directly from the empirical x-ray absorption data. As the Auger rate is too small and no other significant radiationless process exists, it follows that the effect found by Stearns and Stearns must have its origin outside the target. This conclusion is supported by noting that the discrepancies for the μ-K, μ-L, and π-L series, although usually considered separately, are really the same and are a function of only the photon energy, and independent of the target.[2] For example, the targets of boron, fluorine, and sodium give, respectively, μ-K, π-L, and μ-L x rays all with photon energies within 5 kev of 47 kev, and all with a yield of only one-half of the expected yield.

The Auger rates for the μ-K and μ-L series have been computed by Burbidge and de Borde.[3] These rates are too small by factors of 300 and of 30, respectively, to explain the drop in x-ray yield found by Stearns and Stearns. It might be

supposed that somehow the Auger rate calculation could be in error to this extent (although this would be surprising because of its straightforwardness).[4] For this reason we wish to emphasize here the reliability of the theoretical Auger rate, by pointing out the following entirely new way of making the calculation.

The combination of the central Coulomb field of strength Z-1 centered at the nucleus and the perturbing dipole field of the meson is equivalent to a new central Coulomb field shifted in position to the charge center of the nucleon-meson system. Since the charge center oscillates with the transition frequency ω, it is convenient to work in the accelerated frame of reference attached to it. In this frame an oscillating inertia, or gravitational, force acts on the atomic electrons. The effect of this force is the same as that of a uniform fluctuating electric field, allowance being made for the charge-to-mass ratio. Thus the Auger rate is equal to the rate of photoelectric ionization, when the atom is subjected to an "equivalent photon flux" given by

$$I = \frac{c}{2\pi\hbar\omega}\left(\frac{m}{e}\right)^2 \omega^4 (Z-1)^{-2} \sum_f |\vec{r}_{fi}|^2$$

$$= (Z-1)^{-2}\, \sigma_T^{-1} w_R, \qquad (1)$$

where we made use of the standard formula for the radiation rate w_R, and have introduced the Thompson cross section $\sigma_T = (8\pi/3)\,(e^2/mc^2)^2$. $\vec{r}$ is the meson coordinate and the other symbols have their conventional significance. Since the ionization rate is given by multiplying the photon flux by the photoelectric cross section σ_{Z-1}, we find for the ratio of the Auger to radiative rates

$$w_A/w_R = (Z-1)^{-2}\, \sigma_{Z-1}/\sigma_T. \qquad (2)$$

It should be emphasized that despite this equivalence of the Auger rate to the photoeffect, the Auger effect is not in fact due to the conversion of photons, and does not take place in the outer radiation zone of the mesonic atom. It instead takes place in the near zone and is due only to the quasi-static electric field. Indeed, Eq. (2) is valid only when retardation is negligible; but this is true in the cases of interest. Of course, Eq. (2) is also based on treating the meson as a dipole. Since the meson orbits are one to two orders of magnitude smaller than that of the electron, this is a justifiable approximation.

One can use Eq. (2) to calculate the Auger rate directly from x-ray absorption data. Consider the L x ray emitted by a μ meson captured in fluorine, of energy 32 kev. The Z-1 atom, oxygen, has an interpolated mass absorption coefficient[5] of 0.302 at this photon energy, corresponding to a photon cross section of 8.06×10^{-24} cm^2. Subtracting from this eight times the Compton cross section[6] of 0.601×10^{-24} cm^2 we find

$$\sigma_{Z-1} = 3.25\times10^{-24}\ \text{cm}^2,$$

or from Eq. (2)

$$w_A/w_R = 0.075. \qquad (3)$$

This can be compared with the value of Burbidge and de Borde[7] of 0.083. But the latter authors did not allow for the screening of one K electron by the other. The screening is approximately taken into account[8] by further reducing the effective nuclear charge by 5/16, so that the electron density at the nucleus must be reduced by the factor

$$[(Z-1-5/16)/(Z-1)]^3 \approx 1-15/16(Z-1) = 0.88.$$

We thus obtain the theoretical value of 0.073, in good agreement with Eq. (3). As this example demonstrates, Eq. (2), by making use of the empirical x-ray absorption data to avoid the tedious calculations with the free-electron Coulomb wave functions, may have some practical advantage. Equation (2) has the further advantage of automatically including screening and the contribution of the higher electron shells, when these are important.

To continue with the example of fluorine, the μ-L yield found by Stearns and Stearns was only $(24\pm2)\%$. The μ-K series yield for the same atom indicates that approximately 20% of the muons jump directly to the ground state from the $n=3$ and higher shells, so that the L-yield would be expected to be about 80%.[9] To reduce this figure to 24%, the Auger rate would have to be 31 times greater than given by Eq. (3). In view of Eq. (2), this would require a photoelectric cross section greater by the same factor, which is, of course, excluded. On the other hand, as emphasized by Bernstein and Wu,[10] processes involving electrons of neighboring atoms make no significant contribution, since the only essential feature of the initial electron distribution which enters is the density at the nucleus. The further possibility of low-energy radiationless transitions between levels belonging to the same quantum number $n=3$, as suggested by Krall and Gerjuoy[11] for $n=2$, can be ruled out as there is

no metastable state available.[12] In any case, from the existence of the K radiation the mesons are known to have passed through the $n=2$ level. Hence, one can conclude that the L radiation must have been emitted but somehow eluded detection.

To test this hypothesis, it is natural to assume that the discrepancies found with the μ-K, μ-L, and π-L yields, although generally discussed separately, are in fact the same thing. In order to compare the various yield curves, it is necessary to plot them vs energy rather than vs atomic number.[13] Figure 1 shows the data of Stearns and Stearns[1] and of Stearns, Stearns, and Leipuner,[14] plotted in this manner. As a rough approximation all three sets of data, taken with the same apparatus, define the same curve. This indicates a yield independent of the target and depending only upon the quantum energy, or, "by definition," an instrumental effect. A reasonable fit is given by a linear fall-off of detector efficiency with decreasing energy, beginning at about 90 kev. Taking such a linear efficiency and dividing it into the π-K yields found by Stearns and Stearns[15] changes these into a decreasing monotonic function of atomic number, as expected if only nuclear absorption competes with radiation. Furthermore, dividing this linear efficiency function into the roughly constant background found below 90 kev converts it into a bremsstrahlung-type spectrum similar to that found for the background above this energy.[16]

It is well known that the decreased efficiency cannot be attributed to the sodium iodide crystals used for detecting the x rays. Neither can such a linear efficiency curve be attributed to some absorber overlooked between the targets and the detector. Any x-ray absorber which gives a break at 90 kev gives too strong an absorption at the lower energies. But it is sometimes possible for an inefficiency to be introduced by the electronic circuitry for small pulse heights.[17] Finally, note should be taken of the decrease in yield reported in reference 14 for the higher cyclotron intensities. It would be desirable to have further experimental studies of this effect, especially in the crucial energy range of less than 90 kev.

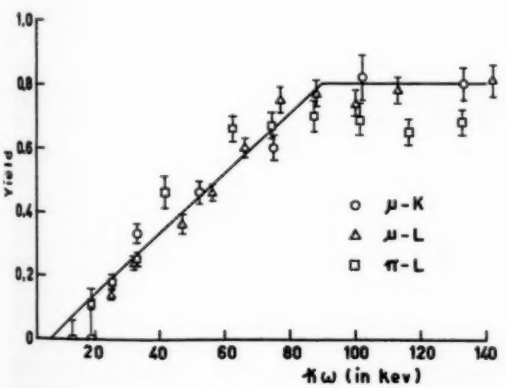

FIG. 1. μ-K, μ-L, and π-L x-ray yields vs photon energy, $\hbar\omega$. The three sets of data, taken with the same experimental apparatus but with different target materials, can be accounted for by the single linear efficiency curve shown.

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*National Science Foundation Senior Postdoctoral Fellow.

[1]M. Stearns and M. B. Stearns, Phys. Rev. 105, 1573 (1951).

[2]This has already been noted by many persons. See, for example, reference 12.

[3]G. R. Burbidge and A. H. de Borde, Phys. Rev. 89, 189 (1953).

[4]The satisfactory agreement found in ordinary atomic applications also makes this unlikely. See E. H. S. Burhop, The Auger Effect (Cambridge University Press, New York, 1952).

[5]A. H. Compton and S. K. Allison, X-Rays in Theory and Experiment (D. van Nostrand Company Inc., New York, 1935), Appendix IX.

[6]W. Heitler, The Quantum Theory of Radiation (Oxford University Press, New York, 1954), 3rd ed., p. 221, Eq. (46).

[7]Since Table I of reference 3 does not include fluorine, we have extrapolated from the μ-L Auger rates for carbon, nitrogen, and oxygen.

[8]L. I. Schiff, Quantum Mechanics (McGraw-Hill Book Company Inc., New York, 1955), 2nd ed., p. 176.

[9]Actually, about one-quarter of the "higher transitions" must be addition peaks, so that the true L yield should be about 85%.

[10]J. Bernstein and T. Y. Wu, Phys. Rev. Letters 2, 404 (1959).

[11]N. A. Krall and E. Gerjuoy, Phys. Rev. Letters 3, 142 (1959).

[12]This mechanism has recently been analyzed in detail by M. A. Ruderman (to be published).

[13]This has already been done by Ruderman, reference 12, Fig. 1. We nevertheless include such a plot here, since a linear rather than logarithmic scale is more convenient for a discussion of the origin of the effect.

[14]M. B. Stearns, M. Stearns, and L. Leipuner, Phys. Rev. 108, 445 (1957).

[15]M. Stearns and M. B. Stearns, Phys. Rev. 107, 1709 (1957).

[16]From later measurements made with the Carnegie Tech. facilities there is good reason to attribute most of the background to bremsstrahlung [G. Backenstoss (private communication)].

[17]It should, however, be noted that checks were made

in the experiments to guard against such a possibility [M. B. Stearns (private communication)]. It should also be noted that the Rochester group independently found a similar decrease in $\pi^- - L$ x-ray yield, but with a somewhat weaker fall-off at the lower energies [M. Camac, M. L. Halbert, and J. B. Platt, Phys. Rev. 99, 905 (1955)]. One can only conclude that similar experimental conditions must have prevailed.

UPPER LIMIT FOR PRODUCTION OF $\Sigma^- n$ HYPERFRAGMENTS BY K^- CAPTURE IN DEUTERIUM*

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(Received March 21, 1960)

Recently, Gondolfi et al. reported evidence for the existence of a bound state of the $\Sigma^- n$ system.[1] Pais and Treiman had already suggested that such a system might be produced when K^- mesons were captured in deuterium.[2] They assumed that the $\Sigma^- n$ would form a deuteron-like structure bound in either the ^1S or ^3S state and used an impulse-type model to estimate the relative rates for the processes

$$K^- + d \rightarrow \Sigma^- n + \pi^+, \quad (1)$$

$$K^- + d \rightarrow \Sigma^- + n + \pi^+, \quad (2)$$

for both odd and even K-Σ parities. Day and Snow have pointed out that the strong S-wave K^--nucleon interaction will dominate the capture process from either S or P atomic orbitals and have extended the calculations of Pais and Treiman to the latter case.[3] Typical values calculated for the fraction of all Σ^- productions leading to the formation of the bound state are summarized in Table I.

In order to determine the relative rates for reactions (1) and (2), we are examining 2100 $\Sigma^-(n)$ productions obtained during two exposures of the Alvarez 15-in. deuterium chamber to the separated 450-Mev/c K^- beam. Of the $\Sigma^-(n)$ productions, $85 \pm 5\%$ result from K^- absorptions at rest. Thus far we have analyzed in detail 227 events in which the $\Sigma^-(n)$ came to rest and was captured via the reactions[4]

$$\Sigma^-(n) + d \rightarrow \Lambda \text{ (or } \Sigma^0) + n + n(+n).$$

For the three cases giving best fits when these events are interpreted as examples of reaction (1), the probabilities for exceeding the observed values of χ^2 were 30%, 10%, and 1%. From this result it is concluded that the fraction of Σ^- productions leading to formation of the bound state is $< 1\%$.

Day, Snow, and Sucher have argued that it is extremely likely that K^- mesons, when stopped in liquid H_2 or D_2, are captured from high-lying S orbitals.[5] If their prediction is correct and if the existence of the $\Sigma^- n$ is confirmed, the present

Table I. Probability for formation of a $\Sigma^- n$ with binding energy E_b when a K^- meson is captured in deuterium via the S-wave K-nucleon interaction.

K-Σ parity	$\Sigma^- n$ bound state	S-orbit capture[a] $E_b = 0.1$ Mev	S-orbit capture[a] $E_b = 0.5$ Mev	P-orbit capture[b] $E_b = 0.1$ Mev	P-orbit capture[b] $E_b = 0.5$ Mev
−	^3S	0.20	0.37	0.03	0.06
−	^1S	forbidden		0	0
+	^3S	0.13	0.25	0.02	0.04
+	^1S	0.07	0.13	0.01	0.02

[a]See reference 2.
[b]See reference 3.

result can be consistent with the calculations of Pais and Treiman only if (a) the binding energy of the $\Sigma^- n$ is exceedingly small ($\lesssim 5$ kev) or (b) the K^--Σ parity is odd so that reaction (1) is forbidden by conservation of angular momentum and parity.

In the absence of any special mechanism, the usual de-excitation processes for the K^--d atom (collisional Auger effect and radiation) will lead to population of the $2P$ atomic orbital from which a large fraction of K^-'s will undergo direct nuclear capture. Since the hyperfragment rates are suppressed by a factor ~10 (see Table I), no strong conclusion could then be drawn from the present experiment.

The following considerations lead us to believe that the $\Sigma^-(n)$ capture events are more favorable than decay events for correct identification of hyperfragments:

(a) Ross has examined the interactions of Σ^- hyperons produced by absorption of stopped K^- mesons in hydrogen.[6] He finds that more than 95% of the Σ^- captures occur at rest. Therefore, it may be assumed that the present analysis is effectively limited to events in which the $\Sigma^-(n)$ does not interact (or decay) before traveling its full range. If reaction (1) is produced by a stopped K^-, the $\Sigma^- n$ will be collinear with the π^+ and have a range of 2.43 ± 0.05 mm. Since the diameter of such a heavily ionizing stub is ~0.3 mm, the usefulness of the collinearity requirement would decrease rapidly if the shorter path lengths of the decay events were included in the analysis.

(b) The distribution in the Σ^--π^+ included angle for reaction (2) peaks at 180°, and many events in which the Σ^- decays within 2.5 mm of the production vertex would be indistinguishable from those of (1). Therefore, the present requirement of unique range provides a strong additional criterion for correct identification.

(c) No corrections are necessary for (i) events in which the $\Sigma^-(n)$ range is poorly determined because of emission of a decay pion at a small angle or (ii) collinear Σ^-'s produced by a K^- absorption in the hydrogen contamination of the bubble chamber.

The film coordinates of all tracks associated with the Σ^--capture events were punched onto IBM cards with the LRL measuring projector and the tracks were spatially reconstructed by means of suitable IBM programs. The distribution in range and included angle for the reconstructed events (with included angle greater than 140 deg and Σ^- range between 1 and 6 mm) is shown in Fig. 1. There is no evidence for increased concentration of events in the region expected for hyperfragments. As a check on the machine measurements, all events were re-examined on the scanning projector and obviously noncollinear events rejected. For the remainder, the $\Sigma^-(n)$ was assumed to be collinear with the π^+ so that the measured values of the π^+ dip and azimuthal angles (determined to ± 0.5 deg) could be used to calculate the lengths expected on the film for a range of 2.43 mm. Comparison of calculated lengths with direct measurements on the film gave results consistent with the χ^2 estimates deduced from the machine measurements.

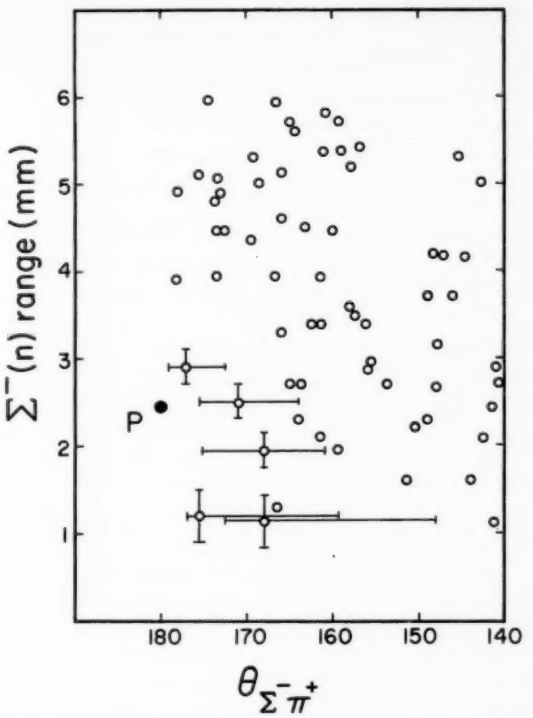

FIG. 1. Measured range vs $\Sigma^-(n)$-π^+ included angle for reactions (1) and (2). The angle and range required for a $\Sigma^- n$ hyperfragment are designated by P. Typical measurement errors are indicated for the six closest events.

The significance of the present measurement depends upon the probability P_c that a $\Sigma^- n$ will come to rest and be captured. This quantity may be written

$$P_c = \exp[-\Gamma_b(T_1 + T_2 + T_c^{D})], \qquad (3)$$

where Γ_b is the decay rate for the bound Σ^-, T_1 is the proper time from production to the time the $\Sigma^- n$ has slowed down to a velocity $v/c = 0.05$ (the approximate lower limit for the validity of the usual range-momentum relation), T_2 is the time from $v/c = 0.05$ to initial capture by a D_2 molecule, and T_C^D is the time from molecular capture to nuclear absorption.

Snow has shown that if charge independence is valid, the $\Sigma^- n$ will probably be bound by less than 0.5 Mev.[7] Dalitz and Liu have argued that for such loosely bound systems, the major modifications in decay rates should arise from the change in phase space available and from the operation of the Pauli principle in the final state.[8] Using Eq. (3.3) given in their paper, we have estimated the magnitude of these effects. Since the S- and P-channel decay amplitudes for the free Σ^- are not known, results for Γ_b are given in Table II for the extreme cases.

Integration of the range-momentum relation yields $T_1 = 0.99 \times 10^{-10}$ sec. Wightman has estimated T_2 for particles of various masses;[9] a slight extrapolation of his results gives $T_2 = 0.19 \times 10^{-10}$ sec. Because of the complexity of the de-excitation process, no reliable estimate is available for T_C; however, some orientation may be gained by consideration of a related process. An equation analogous to Eq. (3) may be used to calculate the capture rate for Σ^- hyperons produced by K^- absorption in hydrogen. The observed capture rate is obtained with $T_C^H = (0.10 \pm 0.25) \times 10^{-10}$ sec.[10] To relate this result to the present situation, we make the rough assumption that the mass dependence is not worse than that characteristic of energy loss to electrons by collision,[11] i.e.,

$$T_C^D \approx (\mu_D/\mu_H) T_C^H \approx (0.2 \pm 0.5) \times 10^{-10} \text{ sec},$$

where μ_D (μ_H) is the reduced mass of the $\Sigma^- n$-d ($\Sigma^- p$) systems.

Under the above assumption, Eq. (3) gives $P_C = 0.32^{+0.17}_{-0.11}$ for the most unfavorable case (K^--Σ parity even, $\Sigma^- n$ bound in the ^1S state). Using this value, we estimate that if the fraction of Σ^- productions leading to the bound state were 1%, 6^{+3}_{-2} $\Sigma^- n$ capture events should have been observed. Since the entire distribution of events is completely consistent with all Σ^-'s being produced via reaction (2), and since, at most, one event can reasonably be interpreted as a hyperfragment production, it is probable that the branching ratio for reaction (1) is substantially less than 1%.

We would like to thank Professor A. Pais and Professor George Snow for informative discussions during the course of this work. It is a pleasure to acknowledge the advice and encouragement of Professor Luis Alvarez and the assistance of the bubble-chamber and scanning staffs of Lawrence Radiation Laboratory.

Table II. Decay rate of $\Sigma^- n$ relative to the free Σ^-. The $\Sigma^- n$ was assumed to be bound by 0.5 Mev.

Amplitude for Σ^- decay into S and P channels	Bound state of $\Sigma^- n$ ^1S	^3S
$S \neq 0$; $P = 0$	1.28	0.94
$S = 0$; $P \neq 0$	0.94	1.04

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] E. Gondolfi, J. Heughebaert, and E. Quercigh, Nuovo cimento 4, 2436 (1959).

[2] A. Pais and A. B. Treiman, Phys. Rev. 107, 1396 (1957).

[3] T. B. Day and G. A. Snow, Phys. Rev. Letters 2, 59 (1959).

[4] O. Dahl, N. Horwitz, D. Miller, and J. Murray, Phys. Rev. Letters 4, 77 (1960).

[5] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters 3, 61 (1959).

[6] Ronald R. Ross, Bull. Am. Phys. Soc. 3, 335 (1958).

[7] George A. Snow, Phys. Rev. 110, 1192 (1958).

[8] R. H. Dalitz and L. Liu, Phys. Rev. 116, 1312 (1959).

[9] A. S. Wightman, Phys. Rev. 77, 521 (1950).

[10] For the free Σ^- decay rate we used $\Gamma_b = (0.642 \pm 0.039) \times 10^{10}$/sec, the weighted average of the values measured in associated production experiments. These values were reported by Luis W. Alvarez at the Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished). The preliminary value, 0.099 ± 0.007, for the Σ^- capture rate was kindly supplied by Dr. Rosenfeld, Dr. Solmitz, and Dr. Tripp of our group.

[11] This is the mass dependence determined by Wightman for T_2 (reference 9). E. Fermi and E. Teller, Phys. Rev. 72, 406 (1947) obtain a similar mass-dependence for particles losing energy by electronic collisions after capture into high-lying atomic orbitals in solids.

SPHERICAL GRAVITATIONAL WAVES*

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This note presents a class of solutions to Einstein's gravitational equations for empty space. Some of the solutions appear to represent a very simple kind of spherical radiation. The metric considered has the form

$$ds^2 = 2d\rho\, d\sigma + (K - 2H\rho - 2m/\rho)d\sigma^2 - \rho^2 p^{-2}\{[d\xi + (\partial q/\partial\eta)d\sigma]^2 + [d\eta + (\partial q/\partial\xi)d\sigma]^2\},$$

where m is a function of σ only, p and q are functions of σ, ξ, η,

$$H = p^{-1}\partial p/\partial\sigma + p\partial^2 p^{-1}q/\partial\xi\partial\eta - pq\partial^2 p^{-1}/\partial\xi\partial\eta,$$

and K is the Gaussian curvature of the surface $\rho = 1$, $\sigma =$ constant,

$$K = p^2(\partial^2/\partial\xi^2 + \partial^2/\partial\eta^2)\ln p.$$

For this metric, the empty-space condition $R_{ik} = 0$ reduces to

$$\partial^2 q/\partial\xi^2 + \partial^2 q/\partial\eta^2 = 0$$

(from which we can derive $q = 0$ by a coordinate transformation), and

$$\partial^2 K/\partial\xi^2 + \partial^2 K/\partial\eta^2 = 4p^{-2}(\partial/\partial\sigma - 3H)m.$$

If these equations are satisfied, the curvature tensor may be written as

$$R_{ijkl} = \rho^{-3}D_{ijkl} + \rho^{-2}\mathrm{III}_{ijkl} + \rho^{-1}N_{ijkl},$$

where D_{ijkl}, III_{ijkl}, and N_{ijkl} are tensors of type I degenerate, type III, and type II null, respectively. They are covariantly constant on any ray of constant σ, ξ, η.

The solution is degenerate type I if m is nonzero and K is independent of ξ, η. It is then reducible to $m = 1$, $p = \cosh\mu\xi$, $q = 0$, where μ is a real or purely imaginary constant. If μ is real and nonzero, this is Schwarzschild's solution for a mass μ^{-3}.

If $(\partial K/\partial\xi)^2 + (\partial K/\partial\eta)^2 \neq 0$ and the empty-space equations are satisfied, the metric is type II nonnull or type III, the condition for type III being $m = 0$. As an example of these types, we mention $m =$ constant, $p = \xi^{3/2}$, $q = 0$. This contains a region of incorrect signature, which can be eliminated, however, by the substitution $\xi = 4\zeta^{-2}$.

In the remaining case, where m vanishes and K is independent of ξ, η, the solution is type II null or flat, the condition for flatness being $A = 0$, where

$$A = (\partial/\partial\xi + i\partial/\partial\eta)[p^2(\partial/\partial\xi + i\partial/\partial\eta)H].$$

For K nonzero, $|A/K\rho|$ is invariant under all coordinate transformations which preserve the form of the metric. It might be described as the intensity of the gravitational field.

This metric and the plane gravitational wave have the same local geometry, since their curvature tensors are algebraically indistinguishable. The wave fronts are subspaces of constant σ. In the linear approximation, they are spheres expanding at the speed of light, the source having a velocity less than, equal to, or greater than this, according as K is positive, zero, or negative.

The coordinates ξ and η may be chosen so that $p = 1 + \frac{1}{4}(\xi^2 + \eta^2)K(\sigma)$. All components of the curvature tensor—covariant, contravariant, and mixed—are then homogeneously linear in q. The contravariant metric tensor may be divided into a flat background metric, independent of q, and a residue which is homogeneously linear in q. There is thus an uncommonly close relation between the rigorous solution and its linear approximation.

It is easy to construct nontrivial solutions which are periodic in σ. In such a solution, there is no monotonic change which could be identified as loss of energy by the source. For positive K, however, there is at least one singularity on any wave front where the field does not vanish identically; and P. G. Bergmann has suggested that this might conceivably represent a flow of matter which restores to the source the energy carried away by radiation.

We can derive the plane-fronted wave,[1]

$$ds^2 = 2d\rho\, d\sigma - 2Hd\sigma^2 - d\xi^2 - d\eta^2,$$

$$\partial H/\partial\rho = \partial^2 H/\partial\xi^2 + \partial^2 H/\partial\eta^2 = 0,$$

from this solution by specializing p as before, substituting $\rho=\lambda^{-2}+\lambda^{-1}\tilde{\rho}$, $\sigma=\lambda\tilde{\sigma}$, $\xi=\lambda^2\tilde{\xi}$, $\eta=\lambda^2\tilde{\eta}$, $q=\lambda^4\tilde{q}$, where λ is constant, and taking the limit as λ tends to zero. There is then a singularity on every wave front where the homogeneity conditions $\partial^3H/\partial\xi^3=\partial^3H/\partial\eta^3=0$ are violated.

We are much indebted to Professor P. G. Bergmann for his comments on an earlier draft of this note.

*This research was supported in part by the U. S. Air Force Office of Scientific Research of the Air Research and Development Command.

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[1]H. W. Brinkmann, Math. Annalen 94, 119 (1925).

POISSON BRACKETS BETWEEN LOCALLY DEFINED OBSERVABLES IN GENERAL RELATIVITY*

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In this note we describe a new method for formulating local observables, and Poisson brackets between them, in Einstein's theory of gravitation. The applicability of this method depends on the functional independence of the four scalars of the Weyl tensor but involves no global or topological assumptions. It combines the Hamiltonian approaches of Dirac[1] with the construction of observables by Komar,[2] and it leads to closed-form expressions both for the observables and the Poisson brackets.

Our point of departure is the discovery that the four scalars of the Weyl tensor can be expressed in closed form in terms of the 12 canonical variables g_{mn}, p^{mn} as defined by Dirac. Using the notation,

$$v_{mn}=(g^{00})^{-1/2}\begin{Bmatrix}0\\m\ n\end{Bmatrix}, \tag{1}$$

to denote Dirac's "invariant velocities," we find for the components of the Weyl tensor the following expressions:

$$\begin{aligned}C_{iklm}&={}^3R_{iklm}+v_{kl}v_{im}-v_{il}v_{km},\\ C_{ikl}&\equiv C_{ikl\rho}l^{\rho}=v_{li/k}-v_{lk/i},\\ C_{kl}&\equiv C_{\rho kl\sigma}l^{\rho}l^{\sigma}=-e^{im}C_{iklm}.\end{aligned} \tag{2}$$

${}^3R_{iklm}$ denotes the three-dimensional curvature tensor, and the solidus signifies covariant differentiation with the help of three-dimensional Christoffel symbols. The Weyl scalars are quadratic and cubic expressions in these components. For instance, the first two (the quadratic) scalars are

$$\begin{aligned}A^1&=C_{iklm}C^{iklm}+4C_{ikl}C^{ikl}+4C_{kl}C^{kl},\\ A^2&=\epsilon^{lms}(C_{iklm}C^{ik}{}_{s}+2C_{klm}C^{k}{}_{s}).\end{aligned} \tag{3}$$

ϵ^{lms} is Levi-Civita's (three-dimensional) fully antisymmetric tensor, and indices are raised and lowered with the help of the three-dimensional metric e^{mn}, g_{mn}.

In order to retain flexibility, we shall assume that the four intrinsic coordinates to be used will be some four functions $f^{\rho}(A^1,\ldots,A^4)$ whose specification we may reserve; if desired they may be chosen so that the f coordinates are asymptotically Lorentzian for some specific Riemann-Einstein manifold. We shall, accordingly, introduce the four coordinate conditions

$$f^{\rho}-x^{\rho}=0, \tag{4}$$

which determine the coordinate system uniquely. These four coordinate conditions, along with the constraints

$$H_S=0,\quad H_L=0, \tag{5}$$

form a system of eight second-class constraints, and hence lend themselves to the construction of field variables and functionals whose Poisson brackets with all the constraints (4), (5) vanish. Such variables are then observables in the technical sense in which this expression is used now in general relativity. They are identical with the observables constructed in reference 2, but now

they appear as closed-form expressions in terms of Dirac's 12 canonical field variables.

As a preliminary we note that the remaining components of the metric tensor are uniquely determined by the coordinate conditions, and that they are closed-form expressions in terms of the canonical variables. We have

$$(g^{00})^{1/2}\delta(x,x') = [f^0, H_L'],$$

$$g^{0n}\delta(x,x') = (g^{00})^{1/2}[f^n, H_L']. \qquad (6)$$

For the consistency of the whole scheme it is of the essence that the Poisson brackets on the right contain only the delta function as a factor, and not, e.g., derivatives of the delta function. This condition is satisfied automatically, because the constraints (5) are generators of infinitesimal coordinate transformations, and the f^ρ, being scalars, have transformation laws that consist solely of the transport term,

$$\bar{\delta} f^\rho = -f^\rho{}_{,\sigma}\,\delta x^\sigma. \qquad (7)$$

Since the right-hand side contains the δx^σ only undifferentiated, no derivatives of the delta function will occur on the right of Eqs. (6).

To construct an observable we start from any functional of the canonical variables, A. We shall now add to A a linear combination of the constraints (4), (5), so that the new functional A^* commutes with all of them,

$$A^* = A + \int [\gamma_\rho (f^\rho - x^\rho) + \epsilon^S H_S + \epsilon^L H_L]\, d^3x. \qquad (8)$$

Provided the constraints are satisfied, A^* will equal in value the original A. The coefficients γ, ϵ are determined as follows:

$$\gamma_k = [H_k, A],$$

$$g^{00}\gamma_0 = (g^{00})^{1/2}[H_L, A] - g^{0s}\gamma_s,$$

$$\epsilon^k = [f^k, A] + \int [f^k, f^{\rho'}]\gamma_\rho' d^3x',$$

$$(g^{00})^{1/2}\epsilon^L = [f^0, A] + \int [f^0, f^{\rho'}]\gamma_\rho' d^3x'. \qquad (9)$$

The inversion of the matrix of the Poisson brackets of the second-class constraints is thus accomplished locally and without difficulty. The Poisson brackets between any two observables A^*, B^* can now be calculated by means of the conventional Poisson brackets between canonical field variables.

The Poisson bracket between A^* and the Hamiltonian vanishes, because the Hamiltonian is but a linear combination of the constraints (5), as was originally shown by Dirac.[1] Nevertheless, A^* is not a constant of the motion, but rather we have

$$\frac{dA^*}{dt} = \frac{\partial A^*}{\partial t} = -\int \gamma_0 d^3x. \qquad (10)$$

A fuller account of our results will be submitted to the Physical Review.

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*Work supported by National Science Foundation and by Wright Air Development Center through Aeronautical Research Laboratory.

[1]P. A. M. Dirac, Can. J. Math. 2, 129 (1950); Proc. Roy. Soc. (London) A246, 333 (1958); Phys. Rev. 114, 924 (1959).

[2]A. Komar, Phys. Rev. 111, 1182 (1958).

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

IRREVERSIBLE THERMODYNAMICS OF A NONLINEAR *R-C* SYSTEM. William Bernard, Research Division, Raytheon Company, Waltham, Massachusetts, and Herbert B. Callen, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received January 26, 1960).

The formalism of nonlinear irreversible thermodynamics, as recently given by us, is applied to a circuit containing a capacitor and a nonlinear resistor. The solution is compared with those obtained by Lax and other investigators. It is shown that the fluctuation-dissipation theorem is rigorous, and that no correction factors need be introduced for nonlinear systems. The dynamical behavior of the microscopic fluctuations, from which the macroscopic motion can be obtained, is also derived. Finally, a specific Markoffian model of a nonlinear *R-C* system, in strong interaction with a temperature reservoir, is shown to be consistent with the general analysis.

IONIC CONDUCTIVITY OF HIGHLY IONIZED PLASMAS. M. Sakuntala, A. von Engel, and R. G. Fowler, Physics Department, University of Oklahoma, Norman, Oklahoma (Received March 30, 1959; revised manuscript received March 17, 1960).

When a cloud of highly ionized gas ejected by a plasma shock tube is made to travel across a constant magnetic field, an electromotive force is produced in the plasma in a direction normal to both the field and the plasma path. Using two probes facing one another this electromotive force has been measured with an oscillograph. Its maximum value was found to be proportional to the field and the probe separation. By taking the maximum probe potential for different values of the external resistance between the probes, the lowest value of the "resistivity of the plasma" as measured by a current entering and leaving it was obtained. The resistivity has been shown to be independent of the magnetic field, the collecting area, the separation and surface state of the probes. All experiments were made in hydrogen at a gas pressure between 0.5 and 5 mm Hg with a nearly critically damped current pulse of order 10^4 amperes lasting for about 6 - 8 μsec and fields < 2000 gauss.

The plasma resistivity between the probes was found to be of the order of 1 ohm cm at a gas pressure of a few mm of Hg. This is about 100 times larger than the electronic resistivity of a fully ionized gas for currents circulating internally. The degree of ionization is thought to be high enough for interaction between charged particles to predominate. The measured values of the plasma resistivity agree with the results obtained from theory based on ionic conduction in the plasma which is here the necessary prerequisite for maintaining the electric neutrality of the moving plasma.

From the measured probe voltage the flow velocity of the plasma was derived. Its variation with gas pressure agrees with shock-wave theory.

MEANING AND INTERPRETATION OF ACOUSTIC MOMENTUM AND ACOUSTIC RADIATION STRESS. E. J. Post, Bell Telephone Laboratories, Murray Hill, New Jersey (Received April 3, 1958; revised manuscript received May 20, 1959).

This paper surveys viewpoints and available procedures of analysis of acoustic radiation stress and the concept of acoustic momentum.

An alternative approach is suggested based on Eckart's principle and Poynting's condition of separate observability of matter and elastic wave motion. The results are consistent with those obtained by means of the adiabatic principle but show discrepancies with the so-called Eulerian approach for transverse waves.

Radiation stress and radiation momentum are typical field concepts needed to complement the continuum description. In a sense they serve to establish consistency with the particle approach, thus filling a need which the conventional theory of elasticity does not satisfy.

The energy-momentum tensor of elastic wave

motion provides a unique example of a physically meaningful space-time covariance in the framework of Galilean kinematics.

OPTICAL PROPERTIES OF COPPER. S. Roberts, General Electric Research Laboratory, Schenectady, New York (Received December 21, 1959).

Optical properties of solid copper have been measured in a wavelength range from 0.365 to 2.5 microns and at temperatures 90°, 300°, and 500°K. Annealed and electropolished specimens were used. Measurements were done in a vacuum of about 2×10^{-5} mm (Hg) and it was observed that the oxide film which normally forms on copper upon exposure to air could be removed by heating in vacuum to a temperature of 500°K. In the wavelength range below 0.6 micron the data confirm the well-known facts relating to a threshold for interband electronic transitions. At longer wavelengths the optical properties are determined almost entirely by free electrons. Deviations from simple theory are partly explained by the anomalous skin effect and it is concluded that the electronic collisions at the metal surface are diffuse. However, it is shown that the anomalous skin effect is not sufficient to explain the observed deviations and that a more complete interpretation ought to consider the nonspherical nature of the Fermi surface and variation of the relaxation time over this surface.

SPECIFIC HEAT OF DILUTE ALLOYS. W. Marshall, Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, England (Received January 4, 1960; revised manuscript received March 1, 1960).

Recently Zimmerman has observed that the addition of Mn to Cu produces a large contribution to the specific heat which, at low temperatures, is linear in temperature and independent of Mn concentration. It is shown that: (1) this remarkable result can be explained in terms of the well-known Ruderman-Kittel-Yosida spin-spin coupling via conduction electrons; (2) the specific heat results of Beck et al. on FeV and FeCr alloys are probably of essentially the same origin as those of Zimmerman on CuMn; and (3) there are serious objections to the mechanism of antiferromagnetism postulated by Overhauser and used by him to explain the specific heat results. In contrast to the Overhauser theory, no new concepts are involved and it is suggested that the large specific heat comes from a small fraction of Mn spins which, because of the random nature of the alloy, happen to be in small effective fields and therefore not strictly aligned. The theory depends on two plausible assumptions which have not, at present, been proved rigorously valid.

NUCLEAR SPIN-LATTICE RELAXATION CAUSED BY PARAMAGNETIC IMPURITIES. W. E. Blumberg, Department of Physics, University of California, Berkeley, California (Received December 24, 1959).

The behavior of the nuclear magnetization has been examined for the transient region immediately following saturation of the nuclear spin system in the case in which the nuclear relaxation is limited by spin diffusion to paramagnetic impurities. Relaxation effects in the absence of diffusion and the presence of rapid diffusion are discussed. An experiment is reported which substantiates the calculations for the case of diffusion-limited relaxation and, in addition, allows a determination of the correlation time of the magnetic moment of the paramagnetic ion and the coefficient of nuclear spin diffusion.

g FACTOR AND DONOR SPIN-LATTICE RELAXATION FOR ELECTRONS IN GERMANIUM AND SILICON. Laura M. Roth, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received January 29, 1960).

The g factors of electrons in Ge and Si are calculated on the basis of the effective-mass approximation. The results are consistent with experimental spin resonance data. The effect is predicted to be anisotropic, with $g_{\parallel}$ less than $g_{\perp}$. This anisotropy introduces a strong interaction between the electron spin and shear waves, for the singlet donor ground state. This interaction can account in order of magnitude for the observed spin-lattice time τ_S for donor electrons in Si at low temperatures, including both a one-phonon process and a two-phonon Raman-type process. The temperature and magnetic field dependence for the two processes

are predicted to be $\tau_S^{-1} \propto TH^4$ and $\tau_S^{-1} \propto T^7H^2$, respectively. The temperature dependence agrees with experiment; however, there are discrepancies in the magnetic field dependence. τ_S is predicted to be anisotropic, an effect that has not as yet been observed.

IRREVERSIBILITY IN THE SUPERCONDUCTING TRANSITION OF LEAD. R. W. Shaw and D. E. Mapother, Physics Department, University of Illinois, Urbana, Illinois (Received January 20, 1960).

The nature and origins of the hysteresis in the magnetic transition of pure superconducting lead have been studied in some detail. The effect can be introduced in nonhysteretic specimens by plastic deformation at liquid helium temperatures. The hysteresis becomes apparent somewhat below T_c and its width increases monotonically with decreasing temperature. Similar effects can be caused by dilute additions of Ca to Pb. Strain-induced hysteresis anneals out near 300°K but temperatures approaching the melting point appear necessary to remove impurity-induced effects. Isothermal resistive measurements show a small fraction of the superconducting phase persisting to fields several hundred gauss above H_c. The residual superconductivity at high fields is increased by plastic deformation and, in general, seems closely related to the hysteresis effect. The observations suggest the existence of a connected network which pervades the entire specimen volume and consists of very small filaments having a critical field exceeding the reversible critical field of bulk lead. The filaments are believed to be associated with defects in the crystalline lattice.

PARAMAGNETIC RESONANCE AND OPTICAL SPECTRA OF DIVALENT IRON IN CUBIC FIELDS. I. THEORY. W. Low and M. Weger, Department of Physics, Hebrew University, Jerusalem, Israel (Received January 22, 1959; revised manuscript received January 20, 1960).

The energy level splittings of the ground state of the d^6 configuration in cubic and axial fields are given. The Zeeman splittings of the various levels are calculated for weak and strong magnetic fields. In the case of tetrahedral symmetry the effect of the perturbations of the odd-parity configurations d^5p and d^5f on the ground state is estimated.

PARAMAGNETIC RESONANCE AND OPTICAL SPECTRA OF DIVALENT IRON IN CUBIC FIELDS. II. EXPERIMENTAL RESULTS. W. Low and M. Weger, Department of Physics, Hebrew University, Jerusalem, Israel (Received January 22, 1959; revised manuscript received January 20, 1960).

The paramagnetic resonance absorption of Fe^{2+} in MgO is observed at $g = 3.428$ and 6.86. The optical absorption line is found at 10 000 cm^{-1}. The paramagnetic resonance spectrum indicates considerable covalent bonding. The origin of the line at 6.86 is discussed.

In tetrahedral ZnS a paramagnetic line is found at $g = 2.25$ and optical absorption at 3 μ and 0.7 μ. Possible explanations of this spectrum are discussed.

A short discussion of the optical absorption spectra of trivalent iron in MgO is presented.

INFLUENCE OF F CENTERS ON THE LATTICE THERMAL CONDUCTIVITY IN LiF. Robert O. Pohl, Laboratory of Atomic and Solid-State Physics, Cornell University, Ithaca, New York (Received January 25, 1960).

The influence of photochemically produced F centers in LiF on the thermal conductivity is investigated, the density of the F centers being determined optically. The F centers decrease the thermal conductivity appreciably at low temperatures. Additive coloration of KCl has a similar effect. The experimental results are quite different from the results obtained in the case of the isotope effect in Ge. Callaway's theory for the lattice thermal conductivity in the presence of point defects cannot explain the present observations satisfactorily, although it is superior to that of Klemens. Two explanations are proposed: (1) The F centers are not randomly distributed (but the presence of clusters of F centers can be excluded in the experiments). (2) The long-range strain field around the F center does not scatter like a point defect. In support of Callaway's theory it is shown that his model can explain an experimental observa-

tion (first reported by Toxen) about the influence of point defects on the thermal conductivity.

THERMAL EQUILIBRIUM OF COLOR CENTERS IN DOPED KCl CRYSTALS. P. Camagni,* S. Ceresara,* and G. Chiarotti, Istituto di Fisica dell'Università, Pavia, Italy (Received March 2, 1959; revised manuscript received June 15, 1959).

In additively colored crystals of KCl:Sr a thermal reaction between F and Z_2 centers is observed. The reaction is reversible and approaches equilibrium in a few hours, in the range of temperatures between 70°C and 170°C. The equilibrium value of the ratio between the concentrations of F and Z_2 centers, (x_F/x_{Z_2}), does not depend upon the total amount of coloration nor upon the past history of the crystal, and for a given impurity content is a function only of temperature. For crystals containing 2.5×10^{-4} molar fractions of Sr, the experimental points fit the equation

$$x_F/x_{Z_2} = 0.8\times10^{-4}\exp(-0.28/kT),\ kT \text{ in ev.}$$

A comparison of the empirical equation with those that can be derived through the application of the mass-action law to some specific models of the $F \rightleftharpoons Z_2$ conversion is discussed.

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MAGNETISM OF EUROPIUM GARNET. W. P. Wolf, Clarendon Laboratory, Oxford, England, and J. H. Van Vleck, Harvard University, Cambridge, Massachusetts (Received January 18, 1960).

The theoretical expressions for the magnetic moment of a trivalent europium ion in a molecular field arising from exchange are applied to Pauthenet's measurements on europium iron garnet. It is a good approximation to assume that the exchange interaction stems entirely from the coupling with the iron atoms, which greatly simplifies the theory since the molecular field on the europium is then an impressed one and does not have to be determined self-consistently. The calculated variation of the magnetization with temperature is in excellent accord with experiment. The magnitude of the exchange interaction is compared with that in the other rare earth iron garnets; it is almost exactly the same as in gadolinium iron garnet.

MAGNETIC SUSCEPTIBILITY OF METALLIC EUROPIUM. R. M. Bozorth, Bell Telephone Laboratories, Murray Hill, New Jersey, and J. H. Van Vleck, Harvard University, Cambridge, Massachusetts (Received January 21, 1960).

The susceptibility of europium is measured from 1.3 to 300°K in fields up to 12 000 oersteds. This metal is found not to be ferromagnetic, but to have at low temperatures a very high paramagnetic susceptibility, about forty times higher than for the free ion or hydrated salts. Variation of susceptibility with field strength is observed below 100°K. The saturation curvature at low temperatures is very large, and practically independent of temperature. The susceptibility at high temperatures is consistent with a divalent model. The magnetic behavior at low temperatures is hard to interpret on the basis of a divalent ion and can be more readily explained with a trivalent one for which the theory is developed. Metallurgical evidence, however, indicates that metallic europium is divalent even at low temperatures. On the other hand, it is generally believed that europium is trivalent in $EuIr_2$, but if one uses the conventional molecular field theory the reported ferromagnetism of $EuIr_2$ cannot be ascribed to a Eu^{+++} with a reasonable value of the exchange field consistent with that in $GdIr_2$.

ELASTIC CONSTANTS OF PALLADIUM FROM 4.2 TO 300°K. J. A. Rayne, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received January 20, 1960).

Elastic-constant measurements have been made on a single crystal of palladium in the temperature range 4.2-300°K. Extrapolation of the data to absolute zero gives

$$c_{11} = (2.341\pm0.027)\times10^{12} \text{ dyne cm}^{-2},$$

$$c_{12} = (1.761\pm0.027)\times10^{12} \text{ dyne cm}^{-2},$$

$$c_{44} = (0.712\pm0.003)\times10^{12} \text{ dyne cm}^{-2}.$$

The corresponding value of Debye temperature is $\theta_0 = 275\pm8$°K, which compares well with the calorimetric figure of $\theta_0 = 274\pm3$°K. Both shear constants show an anomalous temperature de-

pendence. This dependence can be correlated with the temperature variation of the contribution to $C = c_{44}$ and $C' = \frac{1}{2}(c_{11} - c_{12})$, resulting from the presence of holes in the d band of palladium.

SPIN-LATTICE RELAXATION OF SHALLOW DONOR STATES IN Ge AND Si THROUGH A DIRECT PHONON PROCESS. Hiroshi Hasegawa,* Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received January 22, 1960).

The many-valley character of the conduction band edge of germanium and silicon causes an anisotropy of the g shift and of the deformation potential for the conduction electrons. It is shown that the combination of these two effects provides a mechanism for spin-lattice relaxations of the donor spins in germanium and silicon that yields $1/T_S$ proportional to the temperature T and to the fourth power of the static magnetic field H. Using known data about the deformation potential constant, the g shift, the energy of the intervalley splitting, and the elastic constants, the magnitude of T_S is found to be approximately 2×10^{-3} sec for phosphorus donors in germanium, and 1×10^{4} sec for phosphorus donors in silicon. These values refer to $T = 1.25$°K, $H = 3000$ gauss, with the field applied along the [111] axis. Our mechanism fails to give a finite T_S for donors in silicon, when the field is applied along the [100] axis.

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MAGNETIC PROPERTIES OF THULIUM METAL. D. D. Davis and R. M. Bozorth, Bell Telephone Laboratories, Murray Hill, New Jersey (Received January 21, 1960).

Measurements made from 1.3 to 300°K in fields up to 12 000 oe, show that thulium becomes antiferromagnetic on cooling to 60°K and is ferromagnetic below 22°K. In agreement with earlier work the susceptibility between 70 and 300°K obeys the Curie-Weiss law and the effective Bohr magneton number is 7.6, equal to the theoretical value for the trivalent ion core having two holes in the $4f$ shell. The highest observed ferromagnetic moment is only 0.5 Bohr magneton whereas the theoretical value is $gJ = 7$. Large hysteresis and crystal anisotropy are observed at the lowest temperatures. It is suggested that the anisotropy is so large that the magnetization cannot be deviated appreciably from the hexagonal axis even in the higher fields (70 000 oe) used by Henry; he observed 3.4 Bohr magnetons whereas the highest expected for polycrystalline material on this assumption is 3.5.

DETERMINATION OF THE EFFECTIVE SCATTERING MECHANISM PARAMETER OF ELECTRON TRANSPORT THEORY. Theodore C. Harman, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received January 25, 1960).

For Corbino disk geometry, the variation of thermoelectric power, α, with magnetic field strength, H, is a strong function of the scattering and a weak function of the Fermi level. The zero magnetic field thermoelectric power is a strong function of the Fermi level and a weak function of the scattering. Thus, measurements of $\Delta\alpha/\alpha(0)$ versus H and $\alpha(0)$ at a particular impurity concentration and temperature should yield a scattering parameter and a Fermi level separately. The transport equation for $\Delta\alpha/\alpha(0)$ has been derived for the case of a simple spherically symmetric energy band, arbitrary scattering, exact statistics, and Corbino disk geometry. The thermoelectric power increased 55% as the magnetic field increased to 22 kgauss for mercuric selenide. Although there is some evidence which indicates that the simple band form does not occur in HgSe, a preliminary comparison of theory and experiment showed satisfactory agreement.

INTERNAL FIELD EMISSION AT NARROW p-n JUNCTIONS IN INDIUM ANTIMONIDE. A. G. Chynoweth and R. A. Logan, Bell Telephone Laboratories, Murray Hill, New Jersey (Received January 25, 1960).

An experimental study has been made of the field and temperature dependence of internal field emission in narrow p-n junctions in indium antimonide. Relatively good agreement, both qualitative and quantitative, is obtained between the experimental results and the usual expression for the barrier transparency. From studies of Esaki characteristics at low temperatures

and from the observed temperature dependence of the tunnelling current, it is confirmed that the tunnelling transitions do not involve phonons. Also, it is shown that the temperature dependence of the barrier transparency is determined by that of the energy gap at $k=0$.

ELECTRON PARAMAGNETIC RESONANCE OF Fe^{3+} IN TiO_2 (RUTILE). David L. Carter and Akira Okaya, Department of Physics, Columbia University, New York, New York (Received January 20, 1960).

Paramagnetic resonance has been observed from 2 kMc/sec to 110 kMc/sec in Fe^{3+}-doped single crystals of rutile at 78°K, 4.2°K, and 1.4°K. The rather large zero-field splittings measured between the three doublets are 43.3 ±0.1 kMc/sec and 81.3 ± 0.1 kMc/sec. The derived constants for a spin Hamiltonian are $D=20.35\pm0.1$ kMc/sec, $E=2.21\pm0.07$ kMc/sec, $a=1.1\pm0.2$ kMc/sec, $F=-0.5\pm0.3$ kMc/sec, $g=2.000\pm0.005$. The average linewidth is 35 Mc/sec for 2×10^{19} spins per cm^3, and the average spin-lattice relaxation time T_1 at $T=1.4$°K is 4 msec. T_1 still has a very slow inverse temperature dependence even at $T=78$°K. An increase in sensitivity over metal-walled cavity spectrometers was achieved by using the pieces of Fe-doped rutile as the microwave resonators.

RESONANT ELECTRON CAPTURE AND STRIPPING IN MODERATELY LARGE-ANGLE ATOMIC COLLISIONS. F. P. Ziemba,* G. J. Lockwood, G. H. Morgan, and E. Everhart, Physics Department, University of Connecticut, Storrs, Connecticut (Received January 21, 1960).

Differential scattering of ions by atoms in the energy range of 1 kev to 200 kev has been studied. The incident ion, after a single collision which is hard enough to result in a 5° deflection, was analyzed to determine whether it had captured or lost electrons. The angle 5° was chosen as a typical moderately large angle and held fixed as the energy of the incident ion was varied. When the electron capture probability is plotted vs energy, one or more peaks are observed. For the symmetrical case of He^+ on He, seven peaks are clearly outlined. Four peaks appear in the H^+ on He combination and three with H^+ on H_2. Single or double peaks are found in other cases studied which include H^+ on N_2, O_2, air, Ne, Ar, and Kr; H_2^+ on H_2 and He; H_3^+ on H_2 and He; He^+ on Ne, Ar, and Kr; N^+ on Ar; N_2^+ on N_2; Ne^+ on Ne and Ar; Ne^{++} on Ne and Ar; Ar^+ on Ar; and Kr^+ on Kr. For each case the probabilities for electron capture, scattering without change of charge, and various degrees of electron stripping are plotted vs energy. In those cases in which the electron capture probability curve has more than two peaks, these peaks are nearly evenly spaced when the probabilities are plotted vs the time of the interaction. This indicates an electron exchange effect whose period is of the order of 10^{-16} second. In cases where there are many electrons involved in the colliding atoms the phenomenon is more complicated, but vestiges of this resonant exchange are sometimes observed.

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APPROACH TO EQUILIBRIUM IN QUANTAL SYSTEMS: MAGNETIC RESONANCE. A Sher* and H. Primakoff, Washington University, St. Louis, Missouri (Received January 28, 1960).

The present paper presents a derivation of the "master" or Boltzmann "gain-loss" equation from the Schrödinger equation, i.e., a derivation of the equation for the evolution in time of the probabilities of finding a physical system in its various states from the equation for the corresponding probability amplitudes. The "master" equation is derived for an, in effect completely self enclosed, "supersystem," $[A+B]$, consisting of a "system of interest," $[A]$, and a "surroundings," $[B]$, in relatively weak mutual interaction. A discussion is given of the range of validity of the "master" equation for $[A+B]$ and it is shown that the random phase assumption is required for the state vector of $[A+B]$ at the initial time only. The normally microcanonical character of the equilibrium statistical configuration of $[A+B]$ is demonstrated and a treatment is given of exceptional, "extremely quantal-coherent," initial statistical distributions of $[A+B]$ which may evolve away from equilibrium. Derivations are also presented of the "master" equation for $[A]$ and of the "master" equation for an individual particle or quasi-particle $[q]$, within $[A]$; a discussion

of the range of validity of these "master" equations is given and the normally canonical character of the equilibrium statistical configuration of $[A]$ is deduced. General solutions of the "master" equations for $[A+B]$, $[A]$, and $[q]$ are worked out and the relation between the principles of "microscopic reversibility" and "detailed balance" and the nonoscillatory character of the approach to equilibrium are exhibited. A theorem is presented regarding the time variation of the entropy of $[A]$.

As illustrations of the general methods developed, two important processes in magnetic resonance—the time variation of the longitudinal magnetization, $\langle\mu\rangle_t$, and the time variation of the transverse magnetization, $\langle\mu'\rangle_t$,—are discussed in some detail. It is shown that the variation of $\langle\mu\rangle_t$ with t and of $\langle\mu'\rangle_t$ with t for a "nonrigid" lattice can be described by means of the "master" equation for an individual spin $[q]$ and several special cases are discussed on the basis of the evaluation of the appropriate transition probabilities; a comparison with the "spin-temperature" procedure is also appended. On the other hand, it is demonstrated that for a "rigid" lattice no description of the variation of $\langle\mu'\rangle_t$ with t can be given on the basis of a "master" equation; in this case, quantal coherence effects neglected in the derivation of the "master" equation from the Schrödinger equation are vital and $\langle\mu'\rangle_t$ must be evaluated by a rigorous calculation of Trace{[appropriate time-dependent density matrix]μ'}.

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RETARDATION AND K X RAY RELATIVE INTENSITIES.

G. R. Taylor and W. B. Payne, Virginia Polytechnic Institute, Blacksburg, Virginia (Received December 7, 1959).

The matrix elements for electric dipole transitions with retardation have been numerically evaluated for transitions between the 1S and higher discrete states of a single Dirac electron in a Coulomb field. The results are used to calculate the relative intensities of the principal K x-ray lines in elements of high atomic number. Comparison of the retarded and nonretarded matrix elements confirms the earlier result for lead that the effect of retardation is significant in heavy elements only when the total angular momentum of the electron changes. Comparison with the experiments of Beckman indicate that this conclusion is correct, and that in the $K\alpha_2$ to $K\alpha_1$ intensity ratio the effect of retardation is more significant than screening effects.

Q CORRECTION IN THE SPECTRA OF THE IRON GROUP.

Giulio Racah and Yehuda Shadmi, The Hebrew University, Jerusalem, Israel (Received January 29, 1960).

A semiempirical correction term, proportional to the "seniority operator" Q, is introduced into the Slater formulas for the configurations $3d^n4s^2$ of the second spectra of V, Cr, and Fe. The agreement with the experimental data, obtained by least squares, is improved by about 20%. The physical meaning of this correction is discussed.

EXPERIMENTAL VERIFICATION OF THE "INCOHERENT SCATTERING" THEORY FOR THE TRANSPORT OF RESONANCE RADIATION

A. V. Phelps and A. O. McCoubrey,* Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received January 20, 1960).

The predictions of the incoherent scattering theory of the transport of resonance radiation developed by Holstein and by Biberman are shown to agree with laboratory measurements of the decay constants for the intensity of the mercury 2537A resonance radiation following a period of optical excitation. Also, the relative importance of the diffusion of resonance atoms and the escape of resonance radiation as mechanisms for the destruction of mercury atoms in the $^3P_1$ or resonance state are determined from measurements of decay constants as a function of mercury density and cell radius. The experimental results show that diffusion of the resonance atoms is negligible and that the predictions of imprisonment theory are confirmed to within 15%. The experiment sets an upper limit to the diffusion coefficient at unit gas density for atoms in the $^3P_1$ state of 5×10^{17} cm^{-1} sec^{-1} at 340°K, which is consistent with a value of 4×10^{16} cm^{-1} sec^{-1} predicted using the frequency of excitation transfer collisions calculated by Holstein. The success of the theories based on completely incoherent scattering of resonance radiation points to the desirability of including this feature in the treatment of those

astrophysical problems in which the spectral line shape is determined by Doppler broadening or by collision broadening.

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ANALYSIS OF THE TWO-MODE-OF-FISSION HYPOTHESIS. George P. Ford, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received March 16, 1959; revised manuscript received March 3, 1960).

Mass yields for gamma fission of U^{238}, proton fission of Th^{232}, alpha fission of U^{238}, and deuteron fission of normal uranium have been examined in terms of vector spaces for consistency with the hypothesis that for each target and projectile there are two and only two modes of fission. The first three cases are on the whole consistent with the hypothesis. The fourth is not consistent with the hypothesis; the measured yields include some yields from neutron fission. Some consequences of the hypothesis with the added requirement of non-negative yields and non-negative coordinates are derived.

RADIOCHEMICAL STUDIES OF THE (p,pn) REACTION IN COMPLEX NUCLEI IN THE 80-450 Mev RANGE. Herbert P. Yule* and Anthony Turkevich, Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois (Received January 22, 1960).

Excitation functions for the (p,pn) reaction have been determined in the energy range 82 Mev to 426 Mev for the target nuclei F^{19}, Cu^{65}, and Au^{197}. The absolute values are based on the excitation function for the reaction $C^{12}(p,pn)C^{11}$. These excitation functions exhibit a general decrease with energy and the cross sections lie between 122 mb for Au^{196} at 82 Mev and 23 mb for F^{18} at 426 Mev. An excitation function for the reaction $Al^{27}(p,3pn)Na^{24}$ is also presented. The results of Monte Carlo nuclear cascade calculations have been used to predict these (p,pn) excitation functions. The theoretical results are compared with experimental results of this and other reports. Agreement is obtained for the $F^{19}(p,pn)F^{18}$ excitation function and an extrapolation of the Monte Carlo results. The theoretical excitation functions are about one-half the experimental results for the $Cu^{65}(p,pn)Cu^{64}$ reaction and about one-third the experimental results for $Au^{197}(p,pn)Au^{196}$. The effects on the Monte Carlo calculations of a variation in the radius parameter are examined.

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STRIPPING MECHANISM FOR REACTIONS WITH SMALL Q VALUE: THE REACTION $Li^7(d,p)Li^8$. J. P. F. Sellschop, Nuclear Physics Research Unit, University of the Witwatersrand, Johannesburg, South Africa (Received December 8, 1959).

A number of angular distributions of protons from the reaction $Li^7(d,p)Li^8$ have been measured for a range of incident deuteron energies below 2.5 Mev. These agree remarkably well with a simple form of Butler-Born stripping theory, uncorrected for Coulomb and nuclear effects. A description is given for this unusual agreement in terms of the small Q value, -0.188 Mev, for the reaction.

A resonance in the proton yield is found at an incident deuteron energy of 1.4 Mev which has not been observed in measurements of the β yield from this reaction. Angular distributions measured on and around the resonance show no influence of this on the unusually good stripping patterns.

GAMMA RAYS FROM DEUTERON STRIPPING REACTIONS. G. R. Satchler, Oak Ridge National Laboratory, Oak Ridge, Tennessee, and W. Tobocman, The Rice Institute, Houston, Texas (Received January 19, 1960).

The distorted-wave Born approximation is used to calculate the p-γ angular correlation from several deuteron-stripping (d,p) reactions. One $l=2$ and four $l=1$ captures are considered. Optical potentials with rounded edges are used to distort the wave functions. In some cases the correlation is considerably changed from the pattern predicted by the plane wave Born approximation, and the distortion effects are strongly dependent on the direction of the emitted proton, and on the type of distortion assumed. We include a general discussion of the theory of the $(d,p\gamma)$ correlation.

STUDIES OF LOW-LYING LEVELS OF EVEN-EVEN NUCLEI WITH (d,p) AND (d,t) REACTIONS. Bernard L. Cohen and Robert E. Price, Radiation Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania (Received December 3, 1959).

Low-lying states of even-even nuclei in the vibrational region were studied by exciting them with (d,p) and (d,t) reactions. The relative cross sections for exciting ground states (G) bear little relationship to whether they are allowed or forbidden by the simple Mayer-Jensen configurations, so that configuration mixing is generally large. Some of the details of this mixing are obtained. The allowed portions of these cross sections are generally quite close to the single-particle values. First and second excited states are much more strongly exciting than expected theoretically. In Pd^{104}, Pd^{106}, Pt^{194}, and Pt^{198} searches for the triplet in second excited states indicate that its total spacing must be less than 80 kev. New states were found in the triplet region of Cd^{114} and Cd^{112} bringing the total number of known states in these to 5 and 4, respectively; each includes two 0^+ and two 2^+ states. Higher excited states were studied and in almost all cases they occur below the expected position of the third member of the vibrational band; this gives evidence on the size of the energy gap. The location of all 0^+ levels up to 3 Mev is determined for several nuclei. In two cases the 3^+ states required by the Davidow-Filipov theory are found to be 0^+, and in other cases they are not found at all. A number of previously unknown levels are catalogued.

PARAMAGNETIC AND OPTICAL SPECTRA OF YTTERBIUM IN THE CUBIC FIELD OF CALCIUM FLUORIDE. W. Low, Department of Physics, The Hebrew University, Jerusalem, Israel (Received January 28, 1960).

The paramagnetic resonance spectrum of Yb^{3+} in CaF_2 was observed at 20°K and 3 cm wavelength. The spectrum is described by a cubic spin Hamiltonian $\mathcal{H} = g\beta\vec{H}\cdot\vec{S} + A\vec{S}\cdot\vec{I}$ with $g = 3.426 \pm 0.001$, $S = 1/2$, $A^{171} = (886.5 \pm 1.5)\times 10^{-4}$ cm^{-1}, $A^{173} = (243.2 \pm 0.4)\times 10^{-4}$ cm^{-1}, $I^{171} = 1/2$, $I^{173} = 5/2$. The ratio of magnetic moments is $\mu^{173}/\mu^{171} = 1.374_0 \pm 0.005$. The optical spectrum shows lines of 9774 A, 9770 A, 9763 A, and more diffuse and unresolved bands at 9080 and 12730 A. The paramagnetic spectrum is explained as arising from the Γ_7 doublet. The other levels are removed by at least a few cm^{-1} leading to an isotropic g value of 3Λ or 24/7 for the lowest Γ_7 level.

RESPONSE OF NaI(Tl) TO ENERGETIC HEAVY IONS. E. Newman and F. E. Steigert, Yale University, New Haven, Connecticut (Received January 22, 1960).

The light output of NaI(Tl) was measured as a function of energy for ions of He^4, B^{10}, B^{11}, C^{12}, N^{14}, O^{16}, F^{19}, and Ne^{20} from approximately 1.0 to 10.0 Mev/nucleon. The particle energy was varied with absorbing foils and the degraded beam analyzed in a magnetic spectrometer. Light output becomes linear with energy above approximately 6 Mev/amu. There appears to be an odd-even incident charge effect in the saturation value of the differential efficiency of fluorescence, dL/dE.

NUCLEAR SPIN AND HYPERFINE INTERACTION OF In^{113m}. W. J. Childs and L. S. Goodman, Argonne National Laboratory, Lemont, Illinois (Received January 25, 1960).

The hyperfine structure of the 1.7-hr, 393-kev metastable state of In^{113} has been studied by use of the atomic-beam magnetic-resonance technique. The nuclear spin is found to be 1/2 and the magnetic dipole moment to be -0.21050 ± 0.00002 nm, subject to a possible hyperfine anomaly. The hyperfine separation in the atomic $P_{1/2}$ state is measured to be 781.084 ± 0.010 Mc/sec.

ANGULAR DISTRIBUTION OF FRAGMENTS FROM FISSION OF Au^{197} WITH CARBON IONS. Glen E. Gordon, Almon E. Larsh, and Torbjørn Sikkeland, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received January 26, 1960).

The kinetic energy and angular distribution of fragments from fission of Au^{197} with 123- and 93-Mev carbon ions have been determined by observation of the fragments in gas scintillation and solid-state detectors. Between 20 deg and 160 deg in the center-of-mass system, both

angular distributions lie slightly above a $1/\sin\theta$ curve, falling below it beyond those angles. The anisotropies $[\sigma(0°)/\sigma(90°)]$ are 4.7 and 3.8 for 123- and 93-Mev carbon ions, respectively. The most probable fragment kinetic energies in the center-of-mass system are 73 ± 3 Mev and 71 ± 3 Mev.

THEORETICAL PREDICTIONS FOR THE SPECTRA OF THE ODD-MASS XENON AND TELLURIUM ISOTOPES. Norman K. Glendenning, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received December 18, 1959).

The spectra of the isotopes $Te^{123,125}$, $Xe^{127,129}$ are calculated by assuming that the odd neutron, having available the $3s_{1/2}$ and $2d_{3/2}$ states, is coupled to collective surface vibrations of the core. Good agreement is obtained with the known levels in these nuclei using a reasonable value for the coupling parameter. To obtain the agreement, the $d_{3/2} - s_{1/2}$ splitting, ϵ, must be regarded as a function of neutron number. The manner in which ϵ varies, as found in the intermediate-coupling calculation, is compared with the predictions of the pairing correlation theory originally introduced in connection with superconductivity. Agreement as to the general trend is found. This may be regarded to some extent as an indication of the applicability of the pairing correlation theory to nuclear structure calculations.

PARITY-NONCONSERVING INTERNUCLEON POTENTIALS. R. J. Blin-Stoyle,* Department of Physics and Laboratory of Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received January 22, 1960).

The general form that a parity-nonconserving internucleon potential must take because of invariance requirements is obtained. A detailed calculation is then made of the parity-nonconserving potential arising from a self-interacting current description of weak interactions. If the polar vector part of the current ($J_\mu{}^V$) is conserved, then parity nonconservation of the order 1 part in 10^7 ($\mathcal{F} \approx 10^{-7}$) is to be expected in nuclear processes. Failure to observe such an effect would indicate either that $J_\mu{}^V$ is not conserved or that the self-interacting current description is incorrect.

*On sabbatical leave from the Clarendon Laboratory, Oxford, England.

SPINS AND DECAY MODES OF CERTAIN NEUTRON-DEFICIENT SILVER ISOTOPES. O. Ames, A. M. Bernstein, M. H. Brennan, R. A. Haberstroh, and D. R. Hamilton, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received January 22, 1960).

Isotopically enriched Pd foils were bombarded with protons to determine the origin of the 1.2-hr activity which was previously discovered in an even-A Ag isotope. The only appropriate activity was found in Ag^{104} which was observed to have a (69 ± 3)-min predominantly K-capture activity with intense γ rays of 550, 764, and 920 kev. Using the atomic beam magnetic resonance method it was verified that the 69-min activity has $I = 5$. The $I = 2$ resonance in Ag^{104}, when counted in an x-ray detector, was found to decay with a 69-min half-life with a small admixture of a 27-min component. When viewed in a β counter only the 27-min component is observed. These characteristics of the $I = 2$ resonance can be explained by placing the $I = 2$ level above the $I = 5$ level with an appreciable amount of isomeric transition. This interpretation is supported by the observation of feeding in the 920-kev γ ray which only occurs in the decay of the $I = 5$ state.

Further work with isotopically-enriched Pd foils showed γ rays of 120 and 150 kev, and, tentatively, 260 kev which have been assigned to the decay of 59-min Ag^{103}. A positive identification of a (15 ± 2)-min activity in Ag^{102} has been made.

SOME CROSS SECTIONS FOR THE PRODUCTION OF RADIO-NUCLIDES IN THE BOMBARDMENT OF C, N, O, AND Fe BY MEDIUM ENERGY PROTONS. Masatake Honda and Devendra Lal,* Scripps Institution of Oceanography and School of Science and Engineering, University of California, La Jolla, California (Received November 9, 1959; revised manuscript received March 14, 1960).

A number of nuclide formation cross sections

have been measured, using the Berkeley 184-in. cyclotron, to assist in the interpretation of the data on cosmic-ray-produced nuclides in the atmosphere and in iron meteorites.

Cross sections of H^3 and Be^7 have been measured in bombardments of organic targets containing nitrogen and oxygen by protons of energy 225-730 Mev. Semicarbazide (CH_5N_3O) targets were used to obtain cross sections in air nuclei. The targets were prepared by mixing with a few percent of aluminum dust to permit reliable monitoring of the beam. Polyethylene, aluminum lactate, and Plexiglas targets provided elementary cross sections in carbon and oxygen.

The cross sections for the production of the long-lived isotopes $Cl^{36}(3\times10^5$ yr) and $Al^{26}(8\times10^5$ yr) at 730 Mev, and of a number of short-lived radionuclides at 500 and 730 Mev, in iron bombardment by protons have been measured. These data and those of earlier workers suggest some modifications in empirical relations used for predicting spallation cross sections in the case of nuclides close to stability.

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COSMIC-RAY PRODUCTION RATES OF Be^7 IN OXYGEN, AND OF P^{32}, P^{33}, S^{35} IN ARGON AT MOUNTAIN ALTITUDES. Devendra Lal,* James R. Arnold, and Masatake Honda, Scripps Institution of Oceanography and School of Science and Engineering, University of California, La Jolla, California (Received November 9, 1959; revised manuscript received March 14, 1960).

The production rates of radio-isotopes P^{32}, P^{33}, and S^{35} in argon, and of Be^7 in oxygen have been measured by exposing argon and water to cosmic rays at mountain altitudes for periods of two to four months during 1959. The measured values at $\lambda = 51°N$, atmospheric depth 685 g cm^{-2} are 7.6×10^{-6}, 6.2×10^{-6}, 1.4×10^{-5} atom (gram argon)$^{-1}$ sec^{-1} of P^{32}, P^{33}, and S^{35}, respectively, and 9.0×10^{-6} Be^7 atom (gram oxygen)$^{-1}$ sec^{-1}.

Isotope production rates for all regions in the atmosphere have recently been calculated by Lal et al. The measured production rates are higher than the calculated rates by factors of 1.1, 1.8, 1.4, and 1.8 in the case of radio-isotopes Be^7, P^{32}, P^{33}, and S^{35}, respectively. In this comparison, account has been taken of the fact that cosmic-ray intensity has decreased by about 15% since 1948, the time period to which the calculations apply. The measured production rates in oxygen are assumed to apply to air since cross sections for Be^7 formation have been found to be the same in nitrogen and oxygen over most of the energy region of interest.

The procedure used by Lal et al. yields a fairly accurate picture of the variation in production rates with altitude and latitude in the atmosphere. Isotope production rates in all regions of the atmosphere can, therefore, be obtained by normalizing their calculations at the points where our measurements have been made. The calculated production rates of these isotopes in the troposphere, and in the stratosphere corresponding to observed cosmic-ray intensity during 1948-49, are given. The available data on the concentration of the isotopes in rain-water and their averaged yearly deposition rates are compared with their revised production rates. From such a comparison, more definite conclusions can be drawn than hitherto possible in view of the more accurate knowledge of isotope production rates.

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INELASTIC SCATTERING OF HIGH-ENERGY PROTONS EXCITING A COLLECTIVE LEVEL OF THE NUCLEUS. K. Nishimura, Department of Physics, Rutgers University, New Brunswick, New Jersey (Received October 9, 1959).

An analysis is made of the angular distribution and polarization of 185-Mev protons inelastically scattered by carbon. A rough calculation using the distorted-wave Born approximation shows, even though the quantitative agreement is poor, that the 4.4-Mev level of C^{12} may be interpreted as a collective state.

π-NUCLEON PHASE SHIFTS IN THE ENERGY RANGE 350 TO 600 Mev. W. D. Walker, J. Davis, and W. D. Shephard, University of Wisconsin, Madison, Wisconsin (Received January 27, 1960).

This paper contains the results of attempts to obtain a set of phase shifts valid in the energy region just above the 3-3 resonance.

We have found it possible to extend the family of phase shifts found by the workers at Dubna at an energy of about 300 Mev. The features of this set of phase shifts are that α_1, α_{11} are positive and α_{13} is negative and small. We find in

addition that δ_{13} is positive and δ_{15} negative. At 600 Mev the results are consistent with a resonance in the $d_{3/2}$ state. This resonant state, however, decays a sizable fraction of the time into a final state with two π mesons. There are indications that at energies of 400 to 500 Mev most of the single pion production comes from s and $p_{1/2}$ states.

X-RAY YIELDS FROM μ-MESONIC ATOMS. M. A. Ruderman, Department of Physics, University of California, Berkeley, California (Received December 21, 1959).

The interesting suggestion has been made that the rapid drop in the yield of mesonic K x rays in the light elements may be associated with the capture of μ mesons into the metastable $2s$ state. The mechanisms for making transitions from the $2s$ to the $1s$ state and from various p states into the $2s$ state have been investigated in detail for Li, Be, and B. It is found that the paradoxical reduction of K x rays remains unexplained. (1) Stark mixing of the mesonic $2s$ and $2p$ states by the electric fields of the atomic electrons allows "mixed" Auger-radiative transitions to the $1s$ state to compete favorably with radiationless transitions. These mixed transitions give a high-energy x ray and a relatively negligible (10-50 ev) electronic excitation and so contribute to the observed K_α yield. (2) Even if the above "mixed" transitions are ignored, there is no mechanism which gets a large fraction of μ mesons into the $2s$ state that at the same time does not violently contradict both theoretical estimates and observed K x-ray yields from light μ-mesonic atoms.

POLARIZATION IN PION-NUCLEON SCATTERING AND THE SECOND AND THIRD PION-NUCLEON RESONANCES. Michael J. Moravcsik, Lawrence Radiation Laboratory, University of California, Livermore, California (Received January 25, 1960).

The polarization of the recoil nucleon in pion-nucleon scattering is studied from the point of view of providing a means of distinguishing among the various angular momentum assignments proposed for the higher pion-nucleon resonances. It is shown that polarization in this reaction is just as useful a guide as polarization in photoproduction. In particular, a measurement of the polarization at and in the neighborhood of 90° in the energy range between the first and third resonances should give a fairly convincing verification of one or the other of the prevailing assignments.

EXAMPLE OF A SOLUBLE FIELD THEORY WITH FINITE CHARGE RENORMALIZATION. Herbert M. Fried, Department of Physics, University of California, Los Angeles, California (Received January 11, 1960).

A soluble field theory suggested by the Lee and Machida models is described in which coupling constant renormalization arises from a dressed boson and is finite if the contributing fermions are assumed nonrelativistic. For the unrenormalized charge to be real, the renormalized charge must satisfy a certain inequality depending on the boson and fermion mass ratios; if this inequality is violated a single boson ghost state occurs, as expected.

FOLDY TRANSFORMATION IN THE PION-HYPERON SYSTEM. Akira Kanazawa, Department of Physics, Purdue University, Lafayette, Indiana (Received December 15, 1959).

A unitary transformation, which plays the same role as the Foldy transformation in the pion-nucleon system, is constructed for the case where the pion interacts with both Σ and Λ hyperons through γ_5 couplings. The transformation function and the transformed Hamiltonian are very similar to those of the Foldy transformation, in spite of the complexity of our system in isotopic spin space. The application to practical problems is not considered in this paper.

GROUND-STATE ENERGY OF A MANY-FERMION SYSTEM. II. J. M. Luttinger, University of Pennsylvania, Philadelphia, Pennsylvania, and J. C. Ward, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received December 7, 1959).

The perturbation series for the ground-state energy of a many-fermion system is investigated to arbitrary order for the "isotropic" case. This is the case of over-all spherical symmetry,

both in the interaction and in the unperturbed single-particle energies. It is shown that for spin one-half fermions the Brueckner-Goldstone perturbation series is valid to all orders in the perturbation. For spins greater than one-half it is in general incorrect even in the isotropic case, unless the interactions are spin independent.

The discussion to arbitrary order in the interaction is carried out by means of a Feynman-like propagator formalism, which is developed in detail.

FORMAL PARADOX IN QUANTUM ELECTRODYNAMICS. Herbert M. Fried, University of California, Los Angeles, California (Received September 14, 1959; revised manuscript received November 30, 1959).

The formal paradox concerning the vanishing of the photon self-mass, obtained by a mathematically improper manipulation, is examined in Källén's formulation of electrodynamics. It is suggested that this difficulty can be removed, and the formal manipulation retained, by a regularization of the Heisenberg operators. An alternate method of obtaining the spectral function of the photon commutator is described, and a possible consequence of regularization, in connection with the proof of the renormalization constants' divergence, is briefly discussed.

GENERALIZATION OF QUANTUM MECHANICS. T. E. Phipps, Jr., Massachusetts Institute of Technology, Cambridge, Massachusetts (Received May 7, 1957; revised manuscript received January 20, 1960).

The possibility of generalizing quantum mechanics in such a way as to retain its predictive results, while comprehending additional solutions, is examined. It is found that this can be done through a perfected formal correspondence with Hamilton-Jacobi mechanics, by which one is led to consider generalizations of the Heisenberg postulate of the form $p_k q_j - q_j p_k = S\delta_{jk}$, where S is a quantum analog of Hamilton's principal function. The formalism is shown to be equivalent to a simple change in Hamiltonian, with transformed momentum operators satisfying conventional commutation relations, and with an additional relationship involving formal analogs of the classical "initial constants" adjoined. A particular choice of S $(=\hbar/i)$ leads to a theory identical with wave mechanics apart from a constant (unobservable) phase factor on the wave function. The fact that S may possess other, nonconstant values, demonstrated by a specific example, suggests the ability of the mechanical equations to describe a broader class of physical states than has hitherto been investigated.

PION-PION SCATTERING IN THE φ^4 THEORY. M. Baker and F. Zachariasen, Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California (Received December 21, 1959).

Pion-pion scattering has been calculated using the determinantal method, assuming that a relativistic $(\lambda/4)(\varphi_i\varphi_i)^2$ coupling is responsible for the interaction. The scattering amplitude for the individual partial waves is expressed as a ratio of two power series and terms through λ^3 have been kept in each series. Numerical results for the S and P waves have been obtained. λ is adjusted by attempting to fit the electromagnetic structure of nucleons. The best value of λ obtained by this fit is unfortunately so large that the validity of the determinantal approximation is doubtful.

MOMENT OF INERTIA OF SUPERFLUID MANY-FERMION SYSTEMS. Ronald M. Rockmore, Brookhaven National Laboratory, Upton, New York (Received December 15, 1959).

The effects of possible superfluidity on the cranking moment of a large many-fermion system moving under periodic boundary conditions are investigated within the framework of the theory of superconductivity recently formulated by Bogolyubov. The Hamiltonian is initially subjected to Bogolyubov's general unitary transformation. The collective excitations of the fermions are then considered in the usual pair approximation; the appropriate cranking terms are linear in the boson pair operators. On performing a unitary transformation which transforms away these linear terms, one obtains an expression for the moment of inertia of the system which includes both the effects of possible superfluidity and collective excitation. This expression, by virtue of its stationary property with respect to arbitrary variations in the amplitude

associated with the latter unitary transformation, is then utilized as a variational principle for the moment of inertia. For the normal state, the result previously obtained by the author, that the moment of inertia has the rigid value, is rederived in more compact form. For the superfluid state, one finds that collective excitations effect a marked increase in the superfluid moment at intermediate coupling strengths although the resulting moment is still quite small compared to the rigid value. In contrast to the normal state case, where particle-hole pairs play a major role, this increase is almost entirely due to excitations consisting of particle-pairs or hole-pairs. The precise magnitude of the apparent resonance in the moment produced by the d-wave part of the cranking interaction is dependent to some extent on the features of the particle-particle potential which leads to the superfluid state. Variational expressions for the moment are exhibited for both Yukawa and delta-function shell potentials. These results are identical in charged and neutral Fermi systems. A calculation of the cranking moment at finite temperatures is presented in an Appendix along with an interpretation of it in terms of Bardeen's two-fluid model of superconductivity.

PUPPI-STANGHELLINI DISCREPANCY. H. P. Noyes, Lawrence Radiation Laboratory, University of California, Livermore, California, and D. N. Edwards, Nuclear Physics Research Laboratory, University of Liverpool, Liverpool, England (Received December 21, 1959).

In order to establish the extent of the disagreement between the pion-nucleon forward-scattering-amplitude dispersion relations and experiment in a statistical sense, the uncertainty in the dispersion integrals and S-wave scattering lengths is systematically included in the analysis. To accomplish this, we fit the total cross sections below 335 Mev by a Chew-Low P-wave resonance, phenomenologically modified, and calculate the error matrix for the parameters. Our fit to the total cross sections is statistically at least as good as the Anderson parameterization used in previous work. Ignoring forward scattering amplitudes above 220 Mev because of D-wave uncertainties, we still find that there is less than $4\frac{1}{2}\%$ probability that the published data are compatible with a unique value for the pion-nucleon coupling constant f^2, and that no adjustment of the S-wave scattering lengths can remove the discrepancy. However, if the π^- forward-scattering amplitudes measured by Ashkin et al. at 150 and 170 Mev are abandoned in favor of the values recently obtained by Kruse and Arnold at 130 and 152 Mev, the probability rises to 47.2% for our parameterization, or 8.6% for the Anderson parameterization.

Cini et al. have pointed out that the conventional analysis of the low-energy data to obtain the S-wave scattering lengths does not satisfy crossing symmetry, and a reanalysis by Hamilton and Woolcock gives $a_+ = -0.083$, $a_- = 0.088$, rather than the conventional values of -0.110 and 0.077. We obtain some independent evidence in support of this conclusion by using the dispersion relations to determine a_+, a_-, and f^2 simultaneously. We find:

Energy dependence	f^2	a_+	a_-
Anderson	0.075 ± 0.018	-0.086 ± 0.025	0.071 ± 0.020
Modified Chew-Low	0.086 ± 0.019	-0.101 ± 0.026	0.085 ± 0.020

It is clear that a better theoretical description of the energy dependence of the total cross sections will be required before further progress can be made on this problem.

LEVEL STRUCTURE OF NUCLEAR MATTER AND LIQUID He^3. K. A. Brueckner and Toshio Soda, University of California, La Jolla, California, Philip W. Anderson, Bell Telephone Laboratories, Murray Hill, New Jersey, and Pierre Morel, French Embassy, New York, New York (Received January 15, 1960).

Using the K matrix as computed in the study of nuclear matter and liquid He^3 as the effective interaction at the Fermi surface, the possible superfluidity of these systems has been investigated. A theory of the cooperative phenomenon valid for particle-particle interaction in states of arbitrary angular momentum has been developed following the methods of Bardeen, Cooper, and Schrieffer. It is found for states of relative angular momentum other than $l = 0$ that the particle pairs must be correlated with respect to an arbitrary direction in the medium. As a result the change of structure of the Fermi surface is angularly dependent. An energy gap does not occur other than for $l = 0$, the particle excitation

energy vanishing for certain orientations of the momentum. It is also shown that the specific heat shows a discontinuity at the transition temperature, but somewhat different from the case of $l=0$.

Application of these results to liquid He^3 shows that the cooperative effects arise from the interaction in the state with $l=2$, and that the transition temperature is at about 0.1°K. In nuclear matter the $^1S_0$ interaction is very weak and probably repulsive at the Fermi surface, and the attractive $^1D_2$ interaction gives a negligible energy shift. The $^3S_1$ interaction is attractive and in nuclear matter gives a few tenths of an Mev energy gap. These results suggest that in finite nuclei, with pairing of identical nucleons in the same shell, the cooperative effects are not strictly analogous to those in nuclear matter but instead are closely associated with the finite level spacing.